Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.8 Combinations of Functions: Composite Functions

Section Objectives: Students will know how to find arithmetic combinations and compositions of functions.

I. Arithmetic Combinations of Functions (pp. 84−85)
Pace: 5 minutes
Two functions can be combined to create a new function. State the following definition of the arithmetic combinations of functions**.**

Let f and g be functions with overlapping domains. Then for all x common to both domains:

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(fg)(x) = f(x) \bullet g(x)$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, if $g(x)$ is not equal to zero.

Example 1. Given $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$, find the following.

a)

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x^2 + 2x) + (2x + 1)\\ &= x^2 + 4x + 1\end{aligned}$$

b)

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (x^2 + 2x) - (2x + 1)\\ &= x^2 - 1\end{aligned}$$

c)

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x)\\ &= (x^2 + 2x)(2x + 1)\\ &= 2x^3 + 5x^2 + 2x\end{aligned}$$

d)

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}\\ &= \frac{x^2 + 2x}{2x + 1}\end{aligned}$$

$(x \neq -1/2)$

Tip: Remind students to consider the domains of quotients of functions.

II. Composition of Functions (pp. 86−87) Pace: 15 minutes

Tip: This is a difficult topic for most students. Try referring to Figure 1.90 on page 86 of the text, and use number lines instead of “blobs” to make the concept a little less abstract.

- State the following definition.
  The **composition** of the function f with the function g is
  $(f \circ g)(x) = f(g(x))$.
  The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Example 2. Given $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$, find the following.

(a)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x+1) \\ &= (2x+1)^2 + 2(2x+1) \\ &= 4x^2 + 4x + 1 + 4x + 2 \\ &= 4x^2 + 8x + 3\end{aligned}$$

(b)

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(x^2 + 2x) \\ &= 2(x^2 + 2x) + 1 \\ &= 2x^2 + 4x + 1\end{aligned}$$

(c) Using part (b), you can write
$(g \circ f)(3) = 2(3)^2 + 4(3) + 1 = 31$.

Example 3. $h(x) = (x + 1)^2 - x - 1$. Find two functions f and g such that $h(x) = (f \circ g)(x)$.

$g(x) = x + 1$
$f(x) = x^2 - x$

III. Application (p. 88) Pace: 5 minutes

Example 4. A demand function for a certain product is
$p = f(x) = 50 - 0.05x$
where p is the price of the product when x units are sold.
The cost of producing x units is given by
$C(x) = 0.09x + 12{,}000$.

a) Find the revenue function, $R(x) = xf(x)$.
$R(x) = xf(x) = x(50 - 0.05x) = 50x - 0.05x^2$

b) Find the profit function $P(x) = R(x) - C(x)$.
$P(x) = R(x) - C(x) = (50x - 0.05x^2) - (0.09x + 12{,}000)$
$= -0.05x^2 + 49.01x - 12{,}000$

- Assign the *Writing About Mathematics* on page 88 of the text.