Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.9 Inverse Functions

Section Objectives: Students will know how to find inverses of functions graphically and algebraically.

I. Inverse Functions (pp. 93–94) Pace: 10 minutes

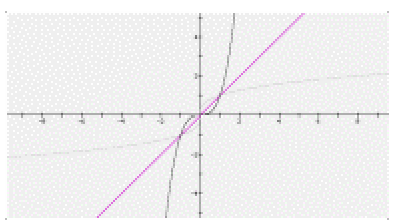
- Relate the concept of inverse functions to multiplication of real numbers, using composition of functions as the operation. So, if we form the composition of two functions, we should get the identity function $h(x) = x$. Inverse functions "undo" each other, so to speak. Also note that if a function is given as a set of ordered pairs, its inverse has all the x-coordinates interchanged with their corresponding y-coordinates.

  Example 1. $f(x) = x + 2$ and $g(x) = x - 2$ are inverses because
  $f(g(x)) = f(x - 2) = (x - 2) + 2 = x$, and
  $g(f(x)) = g(x + 2) = (x + 2) - 2 = x$.

- State the following definition:
  The functions f and g are **inverses** of each other if and only if
  $f(g(x)) = x$ for every x in the domain of g, and
  $g(f(x)) = x$ for every x in the domain of f.
  In this case, we write g as f^{-1}.

II. The Graph of an Inverse Function (p. 95) Pace: 5 minutes

- State that if we interchanged the x's and y's for every point on the graph of a function, the graph would be reflected about the line $y = x$. Illustrate this by using a graphing utility to graph $y = x$, $y = x^3$, and $y = \sqrt[3]{x}$ all on the same viewing screen.



III. One-to-One Functions (p. 96) Pace: 5 minutes

- Draw the graph of $y = x^2$ on the board. Reflect it about the line $y = x$. Ask the class if this inverse is a function. Then ask, "When is the inverse of a function a function?" This brings us to the **Horizontal Line Test**:

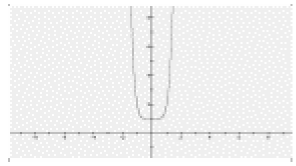
  If every horizontal line meets the graph of a function in at most one point, then the inverse of this function will be a function.

- If no horizontal line intersects the graph of a function in more than one point, no x-value is matched with more than one y-value. This brings us to the definition of a **one-to-one function:**

  A function f is one-to-one if each value of the dependent variable corresponds to exactly one value of the independent variable. A function has an inverse if and only if the function is one-to-one.

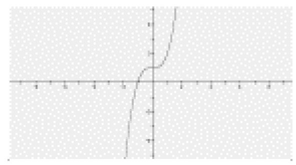
Example 2. Which of the following functions has an inverse function?

(a) No



b)

Yes



IV. Finding Inverse Functions Algebraically (pp. 97–98)
Pace: 10 minutes

- State the following:
  Steps for finding the inverse of a function:
  1. Replace $f(x)$ with y.
  2. Interchange the roles of x and y.
  3. Solve this new equation for y.
  4. Replace y by $f^{-1}(x)$.

Example 3. Find the inverse of each of the following functions.

a)

$$f(x) = \sqrt[3]{x-5}$$
$$y = \sqrt[3]{x-5}$$
$$x = \sqrt[3]{y-5}$$
$$x^3 = y - 5$$
$$y = x^3 + 5$$
$$f^{-1}(x) = x^3 + 5$$

b)

$$g(x) = \frac{x-4}{x+2}$$
$$y = \frac{x-4}{x+2}$$
$$x = \frac{y-4}{y+2}$$
$$xy + 2x = y - 4$$
$$xy - y = -2x - 4$$
$$(x-1)y = -2x - 4$$
$$y = \frac{-2x-4}{x-1}$$
$$g^{-1}(x) = -\frac{2x+4}{x-1}$$