Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.1 Lines

Section Objectives: Students will know how to find the inclination of a line, the angle between two lines, and the distance between a point and a line.

I. Inclination of a Line (pp. 728−729) Pace: 5 minutes

- State that since every nonhorizontal line must meet the x-axis, we will call the angle θ formed by the line and the x-axis ($0 \le \theta < \pi$) the **inclination** of the line.

- State that if a nonvertical line has inclination θ and slope m, then $\tan \theta = m$.

Example 1. Find the inclination of the line given by $2x + y = 5$.
$m = -2$, so $\tan \theta = -2$. $\theta = \tan^{-1}(-2) \approx \pi - 1.1071 \approx 2.0344$

II. The Angle Between Two Lines (pp. 729−730) Pace: 10 minutes

- State that two intersecting lines form two pairs of opposite angles, and the smaller angle is called the **angle between the two lines**. Furthermore, if the two lines have angles of inclination θ_1 and θ_2, then the angle between the two lines is $\theta = \theta_2 - \theta_1$, where $\theta_1 < \theta_2$.

- State that if the two lines have slopes m_1 and m_2, then
$$\tan \theta = \left|\tan(\theta_2 - \theta_1)\right| = \left|\frac{\tan \theta_2 - \tan \theta_1}{1 + \tan \theta_2 \tan \theta_1}\right| = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|.$$

Example 2. Find the angle between the lines given by $3x + 2y = 8$ and $4x - 5y = 1$.
$m_1 = -3/2$ and $m_2 = 4/5$.
$$\tan \theta = \left|\frac{4/5 - (-3/2)}{1 + (4/5)(-3/2)}\right| = \left|-\frac{23}{2}\right|$$
$$\theta = \tan^{-1}\frac{23}{2} \approx 1.4840$$

III. The Distance Between a Point and a Line (pp. 730−731)
Pace: 10 minutes

- State that the distance d between the point (x_1, y_1) and the line $Ax + By + C = 0$ is $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Example 3. Find the distance between the point (2, 3) and the line given by $x - 4y = 0$.
$$d = \frac{|1 \cdot 2 + (-4) \cdot 3 + 0|}{\sqrt{1^2 + (-4)^2}} = \frac{10}{\sqrt{17}}.$$

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Example 4. For the triangle with vertices $A(-1, 2)$, $B(2, 3)$, and $C(1, -1)$, find **(a)** the altitude, and **(b)** the area of the triangle.

(a) We will find the distance from B to the line AC. The equation of the line AC is $3x + 2y + 1 = 0$. So,

$$d = \frac{|3 \cdot 2 + 2 \cdot 3 + 1|}{\sqrt{3^2 + 2^2}} = \frac{13}{\sqrt{13}} = \sqrt{13}.$$

(b) The distance from A to C is the length of the base of the triangle b.

$$b = \sqrt{13}$$

$$A = \frac{1}{2}\sqrt{13}\sqrt{13} = \frac{13}{2}$$

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