Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.2 Introduction to Conics: Parabolas

Section Objectives: Students will know how to recognize a conic, write the standard form of the equation of a parabola, and use the reflective property of parabolas to solve real-life problems.

I. Conics (p. 735) Pace: 5 minutes

- State that a **conic section** is the intersection of a plane and a right circular cone. See Figure 10.9 on page 735 of the text.

II. Parabolas (pp. 736−738) Pace: 20 minutes

- Define a **parabola** as the set of all points (x, y) in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, not on the line. The **vertex** is the midpoint between the focus and the directrix. The **axis** is the line through the focus and perpendicular to the directrix.

- State that the **standard form of the equation of a parabola** with vertex at (h, k), focus at $(p + h, k)$, vertical axis, and directrix $y = k - p$ is $(x - h)^2 = 4p(y - k)$, $p \neq 0$. The standard form of the equation of a parabola with vertex at (h, k), focus at $(h, p + k)$, horizontal axis, and directrix $x = h - p$ is $(y - k)^2 = 4p(x - h)$, $p \neq 0$.

- State that p is the directed distance from the vertex to the focus.

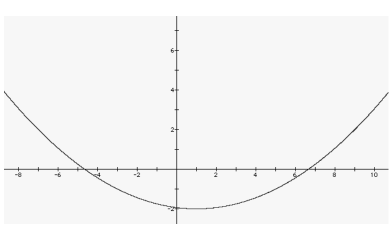
Example 1. Find the vertex, focus, and directrix of the parabola given by $y = 0.5x^2$.
$x^2 = 2y \Rightarrow 4p = 2 \Rightarrow p = \frac{1}{2}$. So, the vertex is at (0, 0), the focus is at (1/2, 0), and the directrix is $y = -1/2$.

Example 2. Find the standard form of the equation of a parabola with vertex at (0, 0) and focus at (0, −2).
$p = -2$ and the axis is horizontal $\Rightarrow y^2 = 4(-2)x$, $y^2 = -8x$.

Example 3. Find the vertex, focus, and directrix of the parabola given by
$x^2 - 2x - 16y - 31 = 0$

$$x^2 - 2x = 16y + 31$$
$$x^2 - 2x + 1 = 16y + 31 + 1$$
$$(x - 1)^2 = 16(y + 2)$$

So, the vertex is at (1, −2) and $p = 4$. Thus the focus is at (1, 2) and the directrix is $y = -6$.



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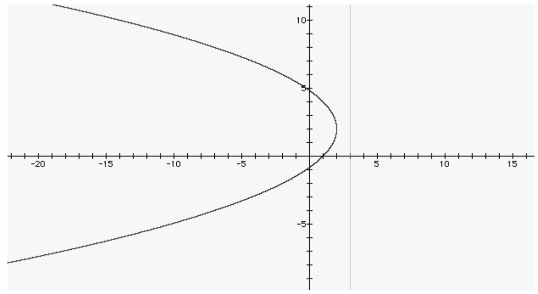
Example 4. Write the standard form of the equation of the parabola with focus at (1, 2) and directrix $x = 3$.
The vertex will be at (2, 2) and $p = -1$. Hence the equation is
$(y - 2)^2 = -4(x - 2)$.



III. Application (pp. 738–739) Pace: 5 minutes

- Discuss the reflective properties of the parabola by describing a television satellite dish. Any satellite signal that comes to the parabola, parallel to its axis, is reflected to the receiver (focus).

- Define the following terms. A line segment with endpoints on a parabola and containing the focus of the parabola is called a **focal chord**. The focal chord perpendicular to the axis of a parabola is called the **latus rectum**. A line is **tangent** to a parabola if it intersects the parabola but does not cross it.

- State that the tangent line to a parabola at a point P makes equal angles with the following two lines.
  1. The line containing the focal chord at P.
  2. The axis of the parabola.

Example 5. Find an equation of the tangent line to the parabola given by $x^2 - 4y = 0$ at the point (4, 4).
Since the vertex is at the origin and $p = 1$, the focus is at (0, 1). The distance from the focus to the point (4, 4) is 5. The distance from the focus to the point where the tangent line meets the y-axis must be the same. Hence the tangent line also passes through (0, −4). Finding the equation of the line through the points (4, 4) and (0, −4) yields
$y = 2x - 4$.