Chapter 1 Practice Test Solutions

1. (a) Midpoint: $\left(\frac{-3+5}{2}, \frac{4+(-6)}{2}\right) = (1, -1)$

(b) Distance: $d = \sqrt{[5-(-3)]^2 + (-6-4)^2}$

$= \sqrt{(8)^2 + (-10)^2}$

$= \sqrt{164} = 2\sqrt{41}$

2. $y = \sqrt{7-x}$

Domain: $x \leq 7$

x	7	6	3	-2
y	0	1	2	3

3. $[x-(-3)]^2 + (y-5)^2 = 6^2$

$(x+3)^2 + (y-5)^2 = 36$

4. $m = \frac{-1-4}{3-2} = -5$

$y - 4 = -5(x-2)$

$y - 4 = -5x + 10$

$y = -5x + 14$

5. $y = \frac{4}{3}x - 3$

6. $2x + 3y = 0$

$y = -\frac{2}{3}x$

$m_1 = -\frac{2}{3}$

$\perp m_2 = \frac{3}{2}$ through (4, 1)

$y - 1 = \frac{3}{2}(x-4)$

$y - 1 = \frac{3}{2}x - 6$

$y = \frac{3}{2}x - 5$

7. (5, 32) and (9, 44)

$m = \frac{44-32}{9-5} = \frac{12}{4} = 3$

$y - 32 = 3(x-5)$

$y - 32 = 3x - 15$

$y = 3x + 17$

When $x = 20$, $y = 3(20) + 17$

$y = \$77.$

8. $f(x-3) = (x-3)^2 - 2(x-3) + 1$

$= x^2 - 6x + 9 - 2x + 6 + 1$

$= x^2 - 8x + 16$

9. $f(3) = 12 - 11 = 1$

$\frac{f(x) - f(3)}{x-3} = \frac{(4x-11)-1}{x-3}$

$= \frac{4x-12}{x-3}$

$= \frac{4(x-3)}{x-3} = 4, x \neq 3$

10. $f(x) = \sqrt{36 - x^2} = \sqrt{(6+x)(6-x)}$

Domain: $[-6, 6]$, because $(6+x)(6-x) \geq 0$ on this interval.

Range: $[0, 6]$, because $0 \leq (6+x)(6-x) \leq 36$ on this interval.

11. (a) $6x - 5y + 4 = 0$

$$y = \frac{6x + 4}{5} \text{ is a function of } x.$$

(b) $x^2 + y^2 = 9$

$$y = \pm\sqrt{9 - x^2} \text{ is not a function of } x.$$

(c) $y^3 = x^2 + 6$

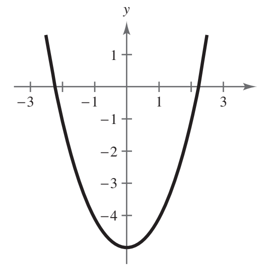
$$y = \sqrt[3]{x^2 + 6} \text{ is a function of } x.$$

12. Parabola

Vertex: $(0, -5)$

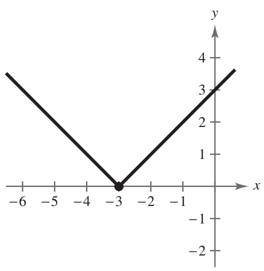
Intercepts: $(0, -5)$, $(\pm\sqrt{5}, 0)$

y-axis symmetry



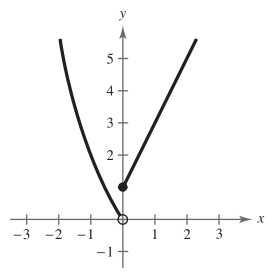
13. Intercepts: $(0, 3)$, $(-3, 0)$

x	0	1	−1	2	−2	−3	−4
y	3	4	2	5	1	0	1



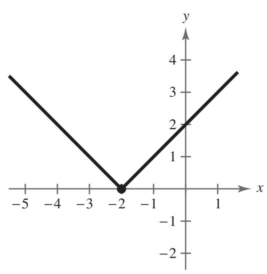
14.

x	0	1	2	3	−1	−2	−3
y	1	3	5	7	2	6	12



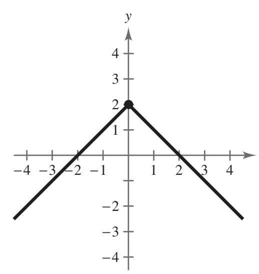
15. (a) $f(x + 2)$

Horizontal shift two units to the left



(b) $-f(x) + 2$

Reflection in the x-axis and a vertical shift two units upward



16. (a)
$$\begin{aligned}(g - f)(x) &= g(x) - f(x) \\ &= (2x^2 - 5) - (3x + 7) \\ &= 2x^2 - 3x - 12\end{aligned}$$

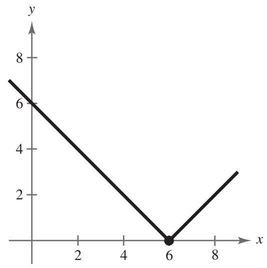
(b)
$$\begin{aligned}(fg)(x) &= f(x)g(x) \\ &= (3x + 7)(2x^2 - 5) \\ &= 6x^3 + 14x^2 - 15x - 35\end{aligned}$$

17.
$$\begin{aligned}f(g(x)) &= f(2x + 3) \\ &= (2x + 3)^2 - 2(2x + 3) + 16 \\ &= 4x^2 + 12x + 9 - 4x - 6 + 16 \\ &= 4x^2 + 8x + 19\end{aligned}$$

18.
$$\begin{aligned}f(x) &= x^3 + 7 \\ y &= x^3 + 7 \\ x &= y^3 + 7 \\ x - 7 &= y^3 \\ \sqrt[3]{x - 7} &= y \\ f^{-1}(x) &= \sqrt[3]{x - 7}\end{aligned}$$

19. (a) $f(x) = |x - 6|$ does not have an inverse.

Its graph does not pass the horizontal line test.



(b) $f(x) = ax + b, a \neq 0$ does have an inverse.

$$y = ax + b$$
$$x = ay + b$$
$$\frac{x - b}{a} = y$$
$$f^{-1}(x) = \frac{x - b}{a}$$

(c) $f(x) = x^3 - 19$ does have an inverse.

$$y = x^3 - 19$$
$$x = y^3 - 19$$
$$x + 19 = y^3$$
$$\sqrt[3]{x + 19} = y$$
$$f^{-1}(x) = \sqrt[3]{x + 19}$$

20. $f(x) = \sqrt{\dfrac{3 - x}{x}},\ 0 < x \leq 3, y \geq 0$

$$y = \sqrt{\frac{3 - x}{x}}$$
$$x = \sqrt{\frac{3 - y}{y}}$$
$$x^2 = \frac{3 - y}{y}$$
$$x^2y = 3 - y$$
$$x^2y + y = 3$$
$$y(x^2 + 1) = 3$$
$$y = \frac{3}{x^2 + 1}$$
$$f^{-1}(x) = \frac{3}{x^2 + 1},\ x \geq 0$$

21. False. The slopes of 3 and $\frac{1}{3}$ are not **negative** reciprocals.

22. True. Let $y = (f \circ g)(x)$. Then $x = (f \circ g)^{-1}(y)$.

Also,

$$(f \circ g)(x) = y$$
$$f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1})(y)$$

Since $x = x$, we have $(f \circ g)^{-1}(y) = (g^{-1} \circ f^{-1})(y)$.

23. True. It must pass the vertical line test to be a function and it must pass the horizontal line test to have an inverse.

24. $z = \dfrac{cx^3}{\sqrt{y}}$

$-1 = \dfrac{c(-1)^3}{\sqrt{25}}$

$-1 = \dfrac{-c}{5}$

$5 = c$

$z = \dfrac{5x^3}{\sqrt{y}}$

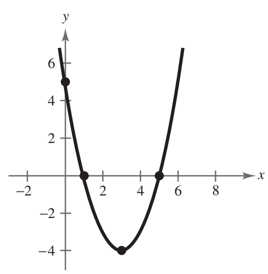
25. $y \approx 0.669x + 2.669$

Chapter 2 Practice Test Solutions

1. x-intercepts: $(1, 0)$, $(5, 0)$

y-intercepts: $(0, 5)$

Vertex: $(3, -4)$



2. $a = 0.01$, $b = -90$

$\dfrac{-b}{2a} = \dfrac{90}{2(.01)} = 4500$ units

3. Vertex: $(1, 7)$ opening downward through $(2, 5)$

$y = a(x - 1)^2 + 7$ Standard form

$5 = a(2 - 1)^2 + 7$

$5 = a + 7$

$a = -2$

$y = -2(x - 1)^2 + 7$

$= -2(x^2 - 2x + 1) + 7$

$= -2x^2 + 4x + 5$

4. $y = \pm a(x - 2)(3x - 4)$ where a is any real number

$y = \pm(3x^2 - 10x + 8)$

5. Leading coefficient: -3

Degree: 5

Moves down to the right and up to the left

6. $0 = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

$= x(x^2 - 1)(x^2 - 4)$

$= x(x + 1)(x - 1)(x + 2)(x - 2)$

$x = 0, x = \pm 1, x = \pm 2$

7. $f(x) = x(x - 3)(x + 2)$

$= x(x^2 - x - 6)$

$= x^3 - x^2 - 6x$

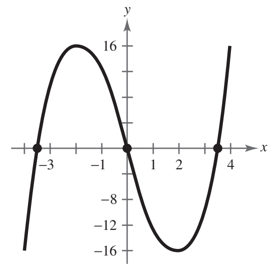
8. Intercepts: $(0, 0), (\pm 2\sqrt{3}, 0)$

Moves up to the right

Moves down to the left

Origin symmetry

x	-2	-1	0	1	2
y	16	11	0	-11	-16



9.

$$\begin{array}{r}
3x^3 + 9x^2 + 20x + 62 + \dfrac{176}{x-3} \\
x - 3 \overline{)\, 3x^4 + 0x^3 - 7x^2 + 2x - 10} \\
\underline{3x^4 - 9x^3} \qquad\qquad\qquad\qquad \\
9x^3 - 7x^2 \qquad\qquad\quad \\
\underline{9x^3 - 27x^2} \qquad\qquad\quad \\
20x^2 + 2x \qquad\quad \\
\underline{20x^2 - 60x} \qquad\quad \\
62x - 10 \\
\underline{62x - 186} \\
176
\end{array}$$

10.

$$\begin{array}{r}
x - 2 + \dfrac{5x - 13}{x^2 + 2x - 1} \\
x^2 + 2x - 1 \overline{)\, x^3 + 0x^2 + 0x - 11} \\
\underline{x^3 + 2x^2 - x} \qquad\quad \\
-2x^2 + x - 11 \\
\underline{-2x^2 - 4x + 2} \\
5x - 13
\end{array}$$

11.

$$\begin{array}{r|rrrrrr}
-5 & 3 & 13 & 0 & 0 & 12 & -1 \\
 & & -15 & 10 & -50 & 250 & -1310 \\
\hline
 & 3 & -2 & 10 & -50 & 262 & -1311
\end{array}$$

$$\frac{3x^5 + 13x^4 + 12x - 1}{x + 5} = 3x^4 - 2x^3 + 10x^2 - 50x + 262 - \frac{1311}{x + 5}$$

12.

$$\begin{array}{r|rrrr}
-6 & 7 & 40 & -12 & 15 \\
 & & -42 & 12 & 0 \\
\hline
 & 7 & -2 & 0 & 15
\end{array}$$

$f(-6) = 15$

13. $0 = x^3 - 19x - 30$

Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$$\begin{array}{r|rrrr}
-2 & 1 & 0 & -19 & -30 \\
 & & -2 & 4 & 30 \\
\hline
 & 1 & -2 & -15 & 0
\end{array}$$

$x = -2$ is a zero.

$0 = (x + 2)(x^2 - 2x - 15)$

$0 = (x + 2)(x + 3)(x - 5)$

Zeros: $x = -2, x = -3, x = 5$

14. $0 = x^4 + x^3 - 8x^2 - 9x - 9$

Possible rational roots: $\pm 1, \pm 3, \pm 9$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -8 & -9 & -9 \\ & & 3 & 12 & 12 & 9 \\ \hline & 1 & 4 & 4 & 3 & 0 \end{array} \quad x = 3 \text{ is a zero.}$$

$0 = (x - 3)(x^3 + 4x^2 + 4x + 3)$

Possible rational roots of $x^3 + 4x^2 + 4x + 3$: $\pm 1, \pm 3$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 4 & 3 \\ & & -3 & -3 & -3 \\ \hline & 1 & 1 & 1 & 0 \end{array} \quad x = -3 \text{ is a zero.}$$

$0 = (x - 3)(x + 3)(x^2 + x + 1)$

The zeros of $x^2 + x + 1$ are $x = \dfrac{-1 \pm \sqrt{3}i}{2}$ (by the Quadratic Formula).

Zeros: $x = 3, x = -3, x = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i, x = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

15. $0 = 6x^3 - 5x^2 + 4x - 15$

Possible rational roots: $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

16. $0 = x^3 - \frac{20}{3}x^2 + 9x - \frac{10}{3}$

$0 = 3x^3 - 20x^2 + 27x - 10$

Possible rational roots:
$\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} 1 & 3 & -20 & 27 & -10 \\ & & 3 & -17 & 10 \\ \hline & 3 & -17 & 10 & 0 \end{array}$$

$0 = (x - 1)(3x^2 - 17x + 10)$
$0 = (x - 1)(3x - 2)(x - 5)$
Zeros: $x = 1, x = \frac{2}{3}, x = 5$

17. Possible rational roots: $\pm 1, \pm 2, \pm 5, \pm 10$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 3 & 5 & -10 \\ & & 1 & 2 & 5 & 10 \\ \hline & 1 & 2 & 5 & 10 & 0 \end{array} \quad x = 1 \text{ is a zero.}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 5 & 10 \\ & & -2 & 0 & -10 \\ \hline & 1 & 0 & 5 & 0 \end{array} \quad x = -2 \text{ is a zero.}$$

$f(x) = (x - 1)(x + 2)(x^2 + 5)$
$= (x - 1)(x + 2)(x + \sqrt{5}i)(x - \sqrt{5}i)$

18. $f(x) = (x - 2)[x - (3 + i)][x - (3 - i)]$

$= (x - 2)[(x - 3) - i][(x - 3) + i]$

$= (x - 2)[(x - 3)^2 - i^2]$

$= (x - 2)[x^2 - 6x + 10]$

$= x^3 - 8x^2 + 22x - 20$

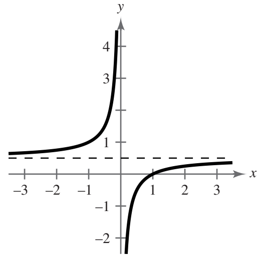
19.

$$\begin{array}{r|rrrr} 3i & 1 & 4 & 9 & 36 \\ & & 3i & 12i - 9 & -36 \\ \hline & 1 & 4 + 3i & 12i & 0 \end{array}$$

20. Vertical asymptote: $x = 0$

Horizontal asymptote: $y = \frac{1}{2}$

x-intercept: $(1, 0)$



21. $y = 8$ is a horizontal asymptote since the degree on the numerator equals the degree of the denominator. There are no vertical asymptotes.

22. $x = 1$ is a vertical asymptote.

$$\frac{4x^2 - 2x + 7}{x - 1} = 4x + 2 + \frac{9}{x - 1}$$

Thus, $y = 4x + 2$ is a slant asymptote.

23. (a) $(4 - 3i) - (-2 + i) = 4 - 3i + 2 - i = 6 - 4i$

(b) $(4 - 3i)(-2 + i) = -8 + 4i + 6i - 3i^2 = -8 + 10i + 3 = -5 + 10i$

(c) $$\frac{4 - 3i}{-2 + i} = \frac{4 - 3i}{-2 + i} \cdot \frac{-2 - i}{-2 - i} = \frac{-8 - 4i + 6i + 3i^2}{4 + 1}$$

$$= \frac{-11 + 2i}{5} = -\frac{11}{5} + \frac{2}{5}i$$

24. $x^2 - 49 \le 0$

$(x + 7)(x - 7) \le 0$

Critical numbers: $x = -7$ and $x = 7$

Test intervals: $(-\infty, -7), (-7, 7), (7, \infty)$

Test: Is $x^2 - 49 \le 0$?

Solution set: $[-7, 7]$

25. $\dfrac{x + 3}{x - 7} \ge 0$

Critical numbers: $x = -3$ and $x = 7$

Test intervals: $(-\infty, -3), (-3, 7), (7, \infty)$

Test: Is $\dfrac{x + 3}{x - 7} \ge 0$?

Solution set: $(-\infty, -3] \cup [7, \infty)$

Chapter 3 Practice Test Solutions

1. $x^{3/5} = 8$

$x = 8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

2. $3^{x-1} = \frac{1}{81}$

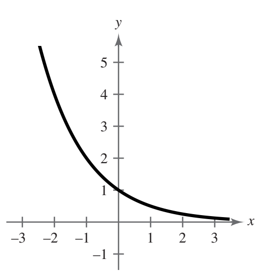
$3^{x-1} = 3^{-4}$

$x - 1 = -4$

$x = -3$

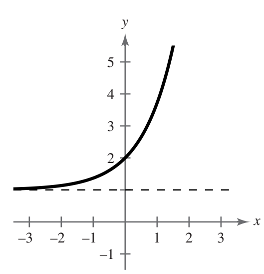
3. $f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$

x	-2	-1	0	1	2
$f(x)$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



4. $g(x) = e^x + 1$

x	-2	-1	0	1	2
$g(x)$	1.14	1.37	2	3.72	8.39



5. (a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{0.09}{12}\right)^{12(3)} \approx \$6543.23$

(b) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{0.09}{4}\right)^{4(3)} \approx \$6530.25$

(c) $A = Pe^{rt}$

$A = 5000e^{(0.09)(3)} \approx \$6549.82$

6. $7^{-2} = \frac{1}{49}$

$\log_7 \frac{1}{49} = -2$

7. $x - 4 = \log_2 \frac{1}{64}$

$2^{x-4} = \frac{1}{64}$

$2^{x-4} = 2^{-6}$

$x - 4 = -6$

$x = -2$

8. $\log_b \sqrt[4]{\frac{8}{25}} = \frac{1}{4} \log_b \frac{8}{25}$

$= \frac{1}{4}[\log_b 8 - \log_b 25]$

$= \frac{1}{4}[\log_b 2^3 - \log_b 5^2]$

$= \frac{1}{4}[3 \log_b 2 - 2 \log_b 5]$

$= \frac{1}{4}[3(0.3562) - 2(0.8271)]$

$= -0.1464$

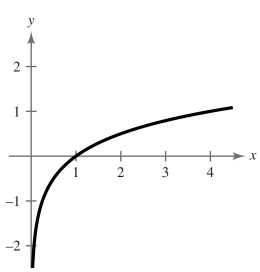
9. $5 \ln x - \frac{1}{2} \ln y + 6 \ln z = \ln x^5 - \ln \sqrt{y} + \ln z^6 = \ln\left(\frac{x^5 z^6}{\sqrt{y}}\right), z > 0$

10. $\log_9 28 = \frac{\log 28}{\log 9} \approx 1.5166$

11. $\log N = 0.6646$

$N = 10^{0.6646} \approx 4.62$

12.



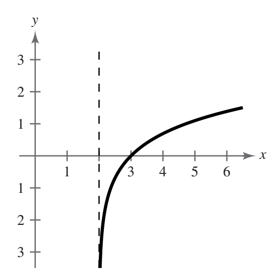
13. Domain:

$x^2 - 9 > 0$

$(x + 3)(x - 3) > 0$

$x < -3$ or $x > 3$

14.



15. False. $\frac{\ln x}{\ln y} \neq \ln(x - y)$ since $\frac{\ln x}{\ln y} = \log_y x$.

16. $5^3 = 41$

$x = \log_5 41 = \frac{\ln 41}{\ln 5} \approx 2.3074$

17. $x - x^2 = \log_5 \frac{1}{25}$

$5^{x-x^2} = \frac{1}{25}$

$5^{x-x^2} = 5^{-2}$

$x - x^2 = -2$

$0 = x^2 - x - 2$

$0 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

18. $\log_2 x + \log_2(x - 3) = 2$

$\log_2[x(x - 3)] = 2$

$x(x - 3) = 2^2$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x = 4$

$x = -1$ (extraneous)

$x = 4$ is the only solution.

19. $\dfrac{e^x + e^{-x}}{3} = 4$

$e^x(e^x + e^{-x}) = 12e^x$

$e^{2x} + 1 = 12e^x$

$e^{2x} - 12e^x + 1 = 0$

$e^x = \dfrac{12 \pm \sqrt{144 - 4}}{2}$

$e^x \approx 11.9161$ or $e^x \approx 0.0839$

$x = \ln 11.9161$ $\quad$ $x = \ln 0.0839$

$x \approx 2.478$ $\quad$ $x \approx -2.478$

20. $A = Pe^{et}$

$12{,}000 = 6000e^{0.13t}$

$2 = e^{0.13t}$

$0.13t = \ln 2$

$t = \dfrac{\ln 2}{0.13}$

$t \approx 5.3319$ years or 5 years 4 months

Chapter 4 Practice Test Solutions

1. $350° = 350\left(\dfrac{\pi}{180}\right) = \dfrac{35\pi}{18}$

2. $\dfrac{5\pi}{9} = \dfrac{5\pi}{9} \cdot \dfrac{180}{\pi} = 100°$

3. $135° \, 14' \, 12'' = \left(135 + \frac{14}{60} + \frac{12}{3600}\right)°$

$\approx 135.2367°$

4. $-22.569° = -(22° + 0.569(60)')$

$= -22° \, 34.14'$

$= -(22° \, 34' + 0.14(60)'')$

$\approx -22° \, 34' \, 8''$

5. $\cos\theta = \dfrac{2}{3}$

$x = 2, r = 3, y = \pm\sqrt{9 - 4} = \pm\sqrt{5}$

$\tan\theta = \dfrac{y}{x} = \pm\dfrac{\sqrt{5}}{2}$

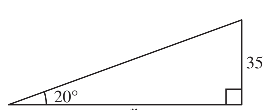
6. $\sin\theta = 0.9063$

$\theta = \arcsin(0.9063)$

$\theta = 65° = \dfrac{13\pi}{36}$ or $\theta = 180° - 65° = 115° = \dfrac{23\pi}{36}$

7. $\tan 20° = \dfrac{35}{x}$

$x = \dfrac{35}{\tan 20°} \approx 96.1617$



8. $\theta = \dfrac{6\pi}{5}$, θ is in Quadrant III.

Reference angle: $\dfrac{6\pi}{5} - \pi = \dfrac{\pi}{5}$ or 36°

9. $\csc 3.92 = \dfrac{1}{\sin 3.92} \approx -1.4242$

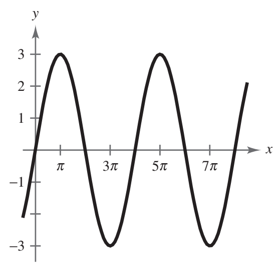
10. $\tan\theta = 6 = \dfrac{6}{1}$, θ lies in Quadrant III.

$y = -6, x = -1, r = \sqrt{36+1} = \sqrt{37}$,

so $\sec\theta = \dfrac{\sqrt{37}}{-1} \approx -6.0828$.

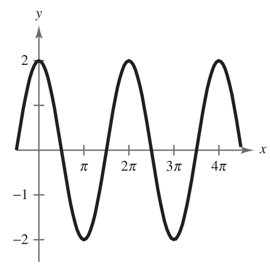
11. Period: 4π

Amplitude: 3

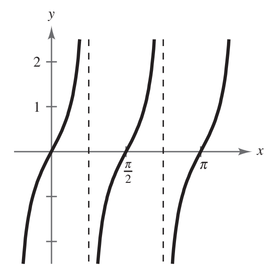


12. Period: 2π

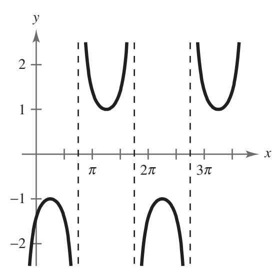
Amplitude: 2



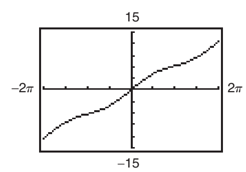
13. Period: $\dfrac{\pi}{2}$



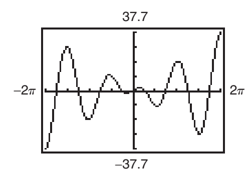
14. Period: 2π



15.



16.



17. $\theta = \arcsin 1$

$\sin\theta = 1$

$\theta = \dfrac{\pi}{2} = 90°$

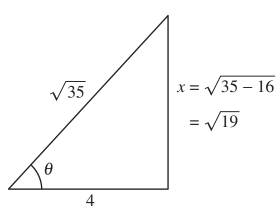
18. $\theta = \arctan(-3)$

$\tan\theta = -3$

$\theta \approx -1.249 \approx -71.565°$

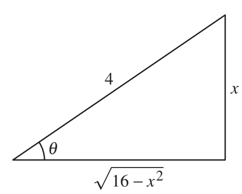
19. $\sin\left(\arccos \dfrac{4}{\sqrt{35}}\right)$

$\sin\theta = \dfrac{\sqrt{19}}{\sqrt{35}} \approx 0.7368$



20. $\cos\left(\arcsin \dfrac{x}{4}\right)$

$\cos\theta = \dfrac{\sqrt{16-x^2}}{4}$



21. Given $A = 40°, c = 12$

$B = 90° - 40° = 50°$

$\sin 40° = \dfrac{a}{12}$

$a = 12 \sin 40° \approx 7.713$

$\cos 40° = \dfrac{b}{12}$

$b = 12 \cos 40° \approx 9.193$

22. Given $B = 6.84°, a = 21.3$

$A = 90° - 6.84° = 83.16°$

$\sin 83.16° = \dfrac{21.3}{c}$

$c = \dfrac{21.3}{\sin 83.16°} \approx 21.453$

$\tan 83.16° = \dfrac{21.3}{b}$

$b = \dfrac{21.3}{\tan 83.16°} \approx 2.555$

23. Given $a = 5, b = 9$

$c = \sqrt{25 + 81} = \sqrt{106} \approx 10.296$

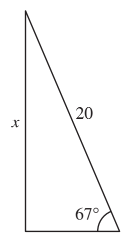
$\tan A = \dfrac{5}{9}$

$A = \arctan \dfrac{5}{9} \approx 29.055°$

$B \approx 90° - 29.055° = 60.945°$

24. $\sin 67° = \dfrac{x}{20}$

$x = 20 \sin 67° \approx 18.41$ feet

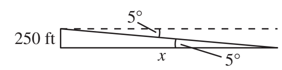


25. $\tan 5° = \dfrac{250}{x}$

$x = \dfrac{250}{\tan 5°}$

≈ 2857.513 feet

≈ 0.541 mi



Chapter 5 Practice Test Solutions

1. $\tan x = \dfrac{4}{11}, \ \sec x < 0 \implies x$ is in Quadrant III.

$y = -4, \ x = -11, \ r = \sqrt{16 + 121} = \sqrt{137}$

$\sin x = -\dfrac{4}{\sqrt{137}} = -\dfrac{4\sqrt{137}}{137}$ $\qquad \csc x = -\dfrac{\sqrt{137}}{4}$

$\cos x = -\dfrac{11}{\sqrt{137}} = -\dfrac{11\sqrt{137}}{137}$ $\qquad \sec x = -\dfrac{\sqrt{137}}{11}$

$\tan x = \dfrac{4}{11}$ $\qquad \cot x = \dfrac{11}{4}$

2.
$$\frac{\sec^2 x + \csc^2 x}{\csc^2 x(1 + \tan^2 x)} = \frac{\sec^2 x + \csc^2 x}{\csc^2 x + (\csc^2 x)\tan^2 x}$$
$$= \frac{\sec^2 x + \csc^2 x}{\csc^2 x + \dfrac{1}{\sin^2 x} \cdot \dfrac{\sin^2 x}{\cos^2 x}}$$
$$= \frac{\sec^2 x + \csc^2 x}{\csc^2 x + \dfrac{1}{\cos^2 x}}$$
$$= \frac{\sec^2 x + \csc^2 x}{\csc^2 x + \sec^2 x} = 1$$

3. $\ln|\tan \theta| - \ln|\cot \theta| = \ln \dfrac{|\tan \theta|}{|\cot \theta|} = \ln\left|\dfrac{\sin \theta/\cos \theta}{\cos \theta/\sin \theta}\right| = \ln\left|\dfrac{\sin^2 \theta}{\cos^2 \theta}\right| = \ln|\tan^2 \theta| = 2 \ln|\tan \theta|$

4. $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{1}{\csc x}$ is true since $\cos\left(\dfrac{\pi}{2} - x\right) = \sin x = \dfrac{1}{\csc x}$.

5. $\sin^4 x + (\sin^2 x)\cos^2 x = \sin^2 x(\sin^2 x + \cos^2 x)$

$= \sin^2 x(1) = \sin^2 x$

6. $(\csc x + 1)(\csc x - 1) = \csc^2 x - 1 = \cot^2 x$

7. $\dfrac{\cos^2 x}{1 - \sin x} \cdot \dfrac{1 + \sin x}{1 + \sin x} = \dfrac{\cos^2 x(1 + \sin x)}{1 - \sin^2 x} = \dfrac{\cos^2 x(1 + \sin x)}{\cos^2 x} = 1 + \sin x$

8. $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}=\dfrac{(1+\cos\theta)^2+\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$$=\frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}=\frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}=\frac{2}{\sin\theta}=2\csc\theta$$

9. $\tan^4 x+2\tan^2 x+1=(\tan^2 x+1)^2=(\sec^2 x)^2=\sec^4 x$

10. (a) $\sin 105^\circ=\sin(60^\circ+45^\circ)=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ$

$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$$

(b) $\tan 15^\circ=\tan(60^\circ-45^\circ)=\dfrac{\tan 60^\circ-\tan 45^\circ}{1+\tan 60^\circ\tan 45^\circ}$

$$=\frac{\sqrt{3}-1}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}=\frac{2\sqrt{3}-1-3}{1-3}=\frac{2\sqrt{3}-4}{-2}=2-\sqrt{3}$$

11. $(\sin 42^\circ)\cos 38^\circ-(\cos 42^\circ)\sin 38^\circ=\sin(42^\circ-38^\circ)=\sin 4^\circ$

12. $\tan\left(\theta+\dfrac{\pi}{4}\right)=\dfrac{\tan\theta+\tan\left(\dfrac{\pi}{4}\right)}{1-(\tan\theta)\tan\left(\dfrac{\pi}{4}\right)}=\dfrac{\tan\theta+1}{1-\tan\theta(1)}=\dfrac{1+\tan\theta}{1-\tan\theta}$

13. $\sin(\arcsin x-\arccos x)=\sin(\arcsin x)\cos(\arccos x)-\cos(\arcsin x)\sin(\arccos x)$

$$=(x)(x)-\left(\sqrt{1-x^2}\right)\left(\sqrt{1-x^2}\right)=x^2-(1-x^2)=2x^2-1$$

14. (a) $\cos(120^\circ)=\cos[2(60^\circ)]=2\cos^2 60^\circ-1=2\left(\dfrac{1}{2}\right)^2-1=-\dfrac{1}{2}$

(b) $\tan(300^\circ)=\tan[2(150^\circ)]=\dfrac{2\tan 150^\circ}{1-\tan^2 150^\circ}=\dfrac{-\dfrac{2\sqrt{3}}{3}}{1-\left(\dfrac{1}{3}\right)}=-\sqrt{3}$

15. (a) $\sin 22.5^\circ=\sin\dfrac{45^\circ}{2}=\sqrt{\dfrac{1-\cos 45^\circ}{2}}=\sqrt{\dfrac{1-\dfrac{\sqrt{2}}{2}}{2}}=\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(b) $\tan\dfrac{\pi}{12}=\tan\dfrac{\frac{\pi}{6}}{2}=\dfrac{\sin\dfrac{\pi}{6}}{1+\cos\left(\dfrac{\pi}{6}\right)}=\dfrac{\dfrac{1}{2}}{1+\dfrac{\sqrt{3}}{2}}=\dfrac{1}{2+\sqrt{3}}=2-\sqrt{3}$

16. $\sin\theta=\dfrac{4}{5}$, θ lies in Quadrant II $\implies$ $\cos\theta=-\dfrac{3}{5}$.

$$\cos\frac{\theta}{2}=\sqrt{\frac{1+\cos\theta}{2}}=\sqrt{\frac{1-\frac{3}{5}}{2}}=\sqrt{\frac{2}{10}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

17. $(\sin^2 x)\cos^2 x = \dfrac{1-\cos 2x}{2}\cdot\dfrac{1+\cos 2x}{2} = \dfrac{1}{4}[1-\cos^2 2x] = \dfrac{1}{4}\left[1-\dfrac{1+\cos 4x}{2}\right]$

$= \dfrac{1}{8}[2-(1+\cos 4x)] = \dfrac{1}{8}[1-\cos 4x]$

18. $6(\sin 5\theta)\cos 2\theta = 6\{\frac{1}{2}[\sin(5\theta+2\theta)+\sin(5\theta-2\theta)]\} = 3[\sin 7\theta + \sin 3\theta]$

19. $\sin(x+\pi)+\sin(x-\pi) = 2\left(\sin\dfrac{[(x+\pi)+(x-\pi)]}{2}\right)\cos\dfrac{[(x+\pi)-(x-\pi)]}{2}$

$= 2\sin x\cos\pi = -2\sin x$

20. $\dfrac{\sin 9x+\sin 5x}{\cos 9x-\cos 5x} = \dfrac{2\sin 7x\cos 2x}{-2\sin 7x\sin 2x} = -\dfrac{\cos 2x}{\sin 2x} = -\cot 2x$

21. $\frac{1}{2}[\sin(u+v)-\sin(u-v)] = \frac{1}{2}\{(\sin u)\cos v+(\cos u)\sin v-[(\sin u)\cos v-(\cos u)\sin v]\}$

$= \frac{1}{2}[2(\cos u)\sin v] = (\cos u)\sin v$

22. $4\sin^2 x = 1$

$\sin^2 x = \dfrac{1}{4}$

$\sin x = \pm\dfrac{1}{2}$

$\sin x = \dfrac{1}{2}$ or $\sin x = -\dfrac{1}{2}$

$x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ $\qquad x = \dfrac{7\pi}{6}$ or $\dfrac{11\pi}{6}$

23. $\tan^2\theta + \left(\sqrt{3}-1\right)\tan\theta - \sqrt{3} = 0$

$(\tan\theta - 1)\left(\tan\theta + \sqrt{3}\right) = 0$

$\tan\theta = 1$ or $\tan\theta = -\sqrt{3}$

$\theta = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$ $\qquad \theta = \dfrac{2\pi}{3}$ or $\dfrac{5\pi}{3}$

24. $\sin 2x = \cos x$

$2(\sin x)\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \dfrac{1}{2}$

$x = \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$ $\qquad x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

25. $\tan^2 x - 6\tan x + 4 = 0$

$\tan x = \dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$

$\tan x = \dfrac{6\pm\sqrt{20}}{2} = 3\pm\sqrt{5}$

$\tan x = 3+\sqrt{5}$ or $\tan x = 3-\sqrt{5}$

$x \approx 1.3821$ or 4.5237 $\qquad x = 0.6524$ or 3.7940

Chapter 6 Practice Test Solutions

1. $C = 180° - (40° + 12°) = 128°$

$a = \sin 40°\left(\dfrac{100}{\sin 12°}\right) \approx 309.164$

$c = \sin 128°\left(\dfrac{100}{\sin 12°}\right) \approx 379.012$

2. $\sin A = 5\left(\dfrac{\sin 150°}{20}\right) = 0.125$

$A \approx 7.181°$

$B \approx 180° - (150° + 7.181°) = 22.819°$

$b = \sin 22.819°\left(\dfrac{20}{\sin 150°}\right) \approx 15.513$

3. Area $= \frac{1}{2}ab \sin C = \frac{1}{2}(3)(6)\sin 130° \approx 6.894$ square units

4. $h = b \sin A = 35 \sin 22.5° \approx 13.394$

$a = 10$

Since $a < h$ and A is acute, the triangle has no solution.

5. $\cos A = \dfrac{(53)^2 + (38)^2 - (49)^2}{2(53)(38)} \approx 0.4598$

$A \approx 62.627°$

$\cos B = \dfrac{(49)^2 + (38)^2 - (53)^2}{2(49)(38)} \approx 0.2782$

$B \approx 73.847°$

$C \approx 180° - (62.627° + 73.847°)$

$= 43.526°$

6. $c^2 = (100)^2 + (300)^2 - 2(100)(300)\cos 29°$

≈ 47522.8176

$c \approx 218$

$\cos A = \dfrac{(300)^2 + (218)^2 - (100)^2}{2(300)(218)} \approx 0.97495$

$A \approx 12.85°$

$B \approx 180° - (12.85° + 29°) = 138.15°$

7. $s = \dfrac{a + b + c}{2} = \dfrac{4.1 + 6.8 + 5.5}{2} = 8.2$

Area $= \sqrt{s(s - a)(s - b)(s - c)}$

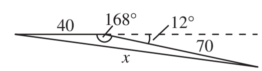
$= \sqrt{8.2(8.2 - 4.1)(8.2 - 6.8)(8.2 - 5.5)}$

≈ 11.273 square units

8. $x^2 = (40)^2 + (70)^2 - 2(40)(70)\cos 168°$

≈ 11977.6266

$x \approx 190.442$ miles



9. $\mathbf{w} = 4(3\mathbf{i} + \mathbf{j}) - 7(-\mathbf{i} + 2\mathbf{j})$

$= 19\mathbf{i} - 10\mathbf{j}$

10. $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{5\mathbf{i} - 3\mathbf{j}}{\sqrt{25 + 9}} = \dfrac{5}{\sqrt{34}}\mathbf{i} - \dfrac{3}{\sqrt{34}}\mathbf{j}$

$= \dfrac{5\sqrt{34}}{34}\mathbf{i} - \dfrac{3\sqrt{34}}{34}\mathbf{j}$

11. $\mathbf{u} = 6\mathbf{i} + 5\mathbf{j} \qquad \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 6(2) + 5(-3) = -3$

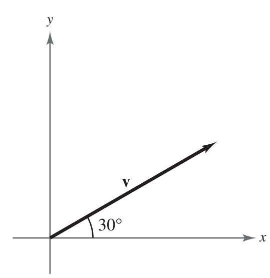
$\|\mathbf{u}\| = \sqrt{61}, \qquad \|\mathbf{v}\| = \sqrt{13}$

$\cos\theta = \dfrac{-3}{\sqrt{61}\sqrt{13}}$

$\theta \approx 96.116°$

12. $4(\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = 4\left(\dfrac{\sqrt{3}}{2}\mathbf{i} + \dfrac{1}{2}\mathbf{j}\right)$

$= \langle 2\sqrt{3}, 2\rangle$



13. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \dfrac{-10}{20}\langle -2, 4\rangle = \langle 1, -2\rangle$

14. $r = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

$\tan\theta = \dfrac{-5}{5} = -1$

Since z is in Quadrant IV, $\theta = 315°$

$z = 5\sqrt{2}(\cos 315° + i \sin 315°)$.

15. $\cos 225° = -\dfrac{\sqrt{2}}{2}, \quad \sin 225° = -\dfrac{\sqrt{2}}{2}$

$z = 6\left(-\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right)$

$= -3\sqrt{2} - 3\sqrt{2}i$

16. $[7(\cos 23° + i \sin 23°)][4(\cos 7° + i \sin 7°)] = 7(4)[\cos(23° + 7°) + i \sin(23° + 7°)]$
$= 28(\cos 30° + i \sin 30°)$

17. $\dfrac{9\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)}{3(\cos \pi + i \sin \pi)} = \dfrac{9}{3}\left[\cos\left(\frac{5\pi}{4} - \pi\right) + i \sin\left(\frac{5\pi}{4} - \pi\right)\right] = 3\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

18. $(2 + 2i)^8 = [2\sqrt{2}(\cos 45° + i \sin 45°)]^8 = \left(2\sqrt{2}\right)^8[\cos(8)(45°) + i \sin(8)(45°)]$
$= 4096[\cos 360° + i \sin 360°] = 4096$

19. $z = 8\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right),\ n = 3$

The cube roots of z are: $\sqrt[3]{8}\left[\cos \dfrac{\frac{\pi}{3} + 2\pi k}{3} + i \sin \dfrac{\frac{\pi}{3} + 2\pi k}{3}\right], k = 0, 1, 2$

For $k = 0$, $\sqrt[3]{8}\left[\cos \dfrac{\frac{\pi}{3}}{3} + i \sin \dfrac{\frac{\pi}{3}}{3}\right] = 2\left(\cos \frac{\pi}{9} + i \sin \frac{\pi}{9}\right)$

For $k = 1$, $\sqrt[3]{8}\left[\cos \dfrac{\left(\frac{\pi}{3}\right) + 2\pi}{3} + i \sin \dfrac{\left(\frac{\pi}{3}\right) + 2\pi}{3}\right] = 2\left(\cos \frac{7\pi}{9} + i \sin \frac{7\pi}{9}\right)$

For $k = 2$, $\sqrt[3]{8}\left[\cos \dfrac{\frac{\pi}{3} + 4\pi}{3} + i \sin \dfrac{\frac{\pi}{3} + 4\pi}{3}\right] = 2\left(\cos \frac{13\pi}{9} + i \sin \frac{13\pi}{9}\right)$

20. $x^4 = -i = 1\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

The fourth roots are: $\sqrt[4]{1}\left[\cos \dfrac{\left(\frac{3\pi}{2}\right) + 2\pi k}{4} + i \sin \dfrac{\left(\frac{3\pi}{2}\right) + 2\pi k}{4}\right], k = 0, 1, 2, 3$

For $k = 0$, $\cos \dfrac{\frac{3\pi}{2}}{4} + i \sin \dfrac{\frac{3\pi}{2}}{4} = \cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}$

For $k = 1$, $\cos \dfrac{\frac{3\pi}{2} + 2\pi}{4} + i \sin \dfrac{\frac{3\pi}{2} + 2\pi}{4} = \cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}$

For $k = 2$, $\cos \dfrac{\frac{3\pi}{2} + 4\pi}{4} + i \sin \dfrac{\frac{3\pi}{2} + 4\pi}{4} = \cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8}$

For $k = 3$, $\cos \dfrac{\frac{3\pi}{2} + 6\pi}{4} + i \sin \dfrac{\frac{3\pi}{2} + 6\pi}{4} = \cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}$

Chapter 7 Practice Test Solutions

1. $\begin{cases} x + y = 1 \\ 3x - y = 15 \implies y = 3x - 15 \end{cases}$

$$\begin{aligned} x + (3x - 15) &= 1 \\ 4x &= 16 \\ x &= 4 \\ y &= -3 \end{aligned}$$

Solution: $(4, -3)$

2. $\begin{cases} x - 3y = -3 \implies x = 3y - 3 \\ x^2 + 6y = 5 \end{cases}$

$$\begin{aligned} (3y - 3)^2 + 6y &= 5 \\ 9y^2 - 18y + 9 + 6y &= 5 \\ 9y^2 - 12y + 4 &= 0 \\ (3y - 2)^2 &= 0 \\ y &= \tfrac{2}{3} \\ x &= -1 \end{aligned}$$

Solution: $\left(-1, \frac{2}{3}\right)$

3. $\begin{cases} x + y + z = 6 \implies z = 6 - x - y \\ 2x - y + 3z = 0 \implies 2x - y + 3(6 - x - y) = 0 \implies -x - 4y = -18 \implies x = 18 - 4y \\ 5x + 2y - z = -3 \implies 5x + 2y - (6 - x - y) = -3 \implies 6x + 3y = 3 \end{cases}$

$$\begin{aligned} 6(18 - 4y) + 3y &= 3 \\ -21y &= -105 \\ y &= 5 \\ x &= 18 - 4y = -2 \\ z &= 6 - x - y = 3 \end{aligned}$$

Solution: $(-2, 5, 3)$

4. $x + y = 110 \implies y = 110 - x$

$$\begin{aligned} xy &= 2800 \\ x(110 - x) &= 2800 \\ 0 &= x^2 - 110x + 2800 \\ 0 &= (x - 40)(x - 70) \end{aligned}$$

$x = 40$ or $x = 70$

$y = 70$ $\quad y = 40$

Solution: The two numbers are 40 and 70.

5. $2x + 2y = 170 \implies y = \dfrac{170 - 2x}{2} = 85 - x$

$$\begin{aligned} xy &= 1500 \\ x(85 - x) &= 1500 \\ 0 &= x^2 - 85x + 1500 \\ 0 &= (x - 25)(x - 60) \end{aligned}$$

$x = 25$ or $x = 60$

$y = 60$ $\quad y = 25$

Dimensions: 60 ft $\times$ 25 ft

6. $\begin{cases} 2x + 15y = 4 \implies 2x + 15y = 4 \\ x - 3y = 23 \implies 5x - 15y = 115 \end{cases}$

$$\begin{aligned} 7x &= 119 \\ x &= 17 \\ y &= \frac{x - 23}{3} \\ &= -2 \end{aligned}$$

Solution: $(17, -2)$

7. $\begin{cases} x + y = 2 \implies 19x + 19y = 38 \\ 38x - 19y = 7 \implies 38x - 19y = 7 \end{cases}$

$$57x = 45$$

$$x = \frac{45}{57} = \frac{15}{19}$$

$$y = 2 - x = \frac{38}{19} - \frac{15}{19} = \frac{23}{19}$$

Solution: $\left(\dfrac{15}{19}, \dfrac{23}{19}\right)$

8. $\begin{cases} 0.4x + 0.5y = 0.112 \implies 0.28x + 0.35y = 0.0784 \\ 0.3x - 0.7y = -0.131 \implies 0.15x - 0.35y = -0.0655 \end{cases}$

$$0.43x = 0.0129$$

$$x = \frac{0.0129}{0.43} = 0.03$$

$$y = \frac{0.112 - 0.4x}{0.5} = 0.20$$

Solution: $(0.03, 0.20)$

9. Let x = amount in 11% fund and y = amount in 13% fund.

$$x + y = 17000 \implies y = 17000 - x$$

$$0.11x + 0.13y = 2080$$

$$0.11x + 0.13(17000 - x) = 2080$$

$$-0.02x = -130$$

$$x = \$6500 \quad \text{at } 11\%$$

$$y = \$10{,}500 \text{ at } 13\%$$

10. $(4, 3), (1, 1), (-1, -2), (-2, -1)$

Use a calculator.

$y = ax + b = \frac{11}{14}x - \frac{1}{7}$

11. $\begin{cases} x + y = -2 \\ 2x - y + z = 11 \\ 4y - 3z = -20 \end{cases}$

$\begin{cases} x + y = -2 \\ -3y + z = 15 \\ 4y - 3z = -20 \end{cases}$ $-2\text{Eq.1} + \text{Eq.2}$

$\begin{cases} x + y = -2 \\ y - 2z = -5 \\ 4y - 3z = -20 \end{cases}$ $\text{Eq.3} + \text{Eq.2}$

$\begin{cases} x + y = -2 \\ y - 2z = -5 \\ 5z = 0 \end{cases}$ $-4\text{Eq.2} + \text{Eq.3}$

$\begin{cases} x + y = -2 \\ y - 2z = -5 \\ z = 0 \end{cases}$

$$y - 2(0) = -5 \implies y = -5$$

$$x + (-5) = -2 \implies x = 3$$

Solution: $(3, -5, 0)$

12. $\begin{cases} 4x - y + 5z = 4 \\ 2x + y - z = 0 \\ 2x + 4y + 8z = 0 \end{cases}$

$\begin{cases} 2x + 4y + 8z = 0 \\ 2x + y - z = 0 \\ 4x - y + 5z = 4 \end{cases}$ Interchange equations.

$\begin{cases} 2x + 4y + 8z = 0 \\ -3y - 9z = 0 & -\text{Eq.1} + \text{Eq.2} \\ -9y - 11z = 4 & -2\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} 2x + 4y + 8z = 0 \\ -3y - 9z = 0 \\ 16z = 4 & -3\text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + 2y + 4z = 0 & \frac{1}{2}\text{Eq.1} \\ y + 3z = 0 & -\frac{1}{3}\text{Eq.2} \\ z = \frac{1}{4} & \frac{1}{16}\text{Eq.3} \end{cases}$

$$y + 3\left(\tfrac{1}{4}\right) = 0 \implies y = -\tfrac{3}{4}$$

$$x + 2\left(-\tfrac{3}{4}\right) + 4\left(\tfrac{1}{4}\right) = 0 \implies x = \tfrac{1}{2}$$

Solution: $\left(-\frac{1}{2}, -\frac{3}{4}, \frac{1}{4}\right)$

13. $\begin{cases} 3x + 2y - z = 5 \\ 6x - y + 5z = 2 \end{cases}$

$\begin{cases} 3x + 2y - z = 5 \\ -5y + 7z = -8 & -2\text{Eq.1} + \text{Eq.2} \end{cases}$

$\begin{cases} x + \frac{2}{3}y - \frac{1}{3}z = \frac{5}{3} & \frac{1}{3}\text{Eq.1} \\ y - \frac{7}{5}z = \frac{8}{5} & -\frac{1}{5}\text{Eq.2} \end{cases}$

Let $a = z$.

Then $y = \frac{7}{5}a + \frac{8}{5}$, and

$$x + \tfrac{2}{3}\left(\tfrac{7}{5}a + \tfrac{8}{5}\right) - \tfrac{1}{3}a = \tfrac{5}{3}$$

$$x + \tfrac{3}{5}a = \tfrac{3}{5}$$

$$x = -\tfrac{3}{5}a + \tfrac{3}{5}.$$

Solution: $\left(-\frac{3}{5}a + \frac{3}{5}, \frac{7}{5}a + \frac{8}{5}, a\right)$, where a is any real number.

14. $y = ax^2 + bx + c$ passes through $(0, -1)$, $(1, 4)$, and $(2, 13)$.

$$\begin{array}{llll}
\text{At } (0, -1)\text{: } -1 = a(0)^2 + b(0) + c & \Rightarrow \quad c = -1 & & \\
\text{At } (1, 4)\text{: } 4 = a(1)^2 + b(1) - 1 & \Rightarrow \quad 5 = a + b & \Rightarrow & 5 = a + b \\
\text{At } (2, 13)\text{: } 13 = a(2)^2 + b(2) - 1 & \Rightarrow 14 = 4a + 2b & \Rightarrow & -7 = -2a - b \\
 & & & -2 = -a \\
 & & & a = 2 \\
 & & & b = 3
\end{array}$$

Thus, the equation of the parabola is $y = 2x^2 + 3x - 1$.

15. $s = \frac{1}{2}at^2 + v_0t + s_0$ passes through $(1, 12)$, $(2, 5)$, and $(3, 4)$.

At $(1, 12)$: $12 = \frac{1}{2}a + v_0 + s_0$

At $(2, 5)$: $5 = 2a + 2v_0 + s_0$

At $(3, 4)$: $4 = \frac{9}{2}a + 3v_0 + s_0$

$$\begin{cases} a + 2v_0 + 2s_0 = 24 \\ 2a + 2v_0 + s_0 = 5 \\ 9a + 6v_0 + 2s_0 = 8 \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 24 & \\ -2v_0 - 3s_0 = -43 & -2\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 = -208 & -9\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 24 & \\ -2v_0 - 3s_0 = -43 & \\ 2s_0 = 50 & -6\text{Eq.2} + \text{Eq.3} \end{cases}$$

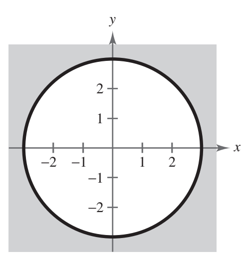
$$\begin{cases} a + 2v_0 + 2s_0 = 24 & \\ v_0 + \frac{3}{2}s_0 = \frac{43}{2} & -\frac{1}{2}\text{Eq.2} \\ s_0 = 25 & \frac{1}{2}\text{Eq.3} \end{cases}$$

$$v_0 + \tfrac{3}{2}(25) = \tfrac{43}{2} \Rightarrow v_0 = -16$$

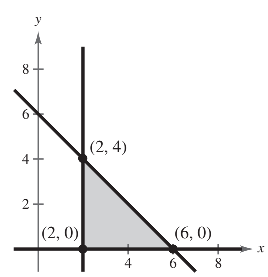
$$a + 2(-16) + 2(25) = 24 \Rightarrow a = 6$$

Thus, $s = \frac{1}{2}(6)t^2 - 16t + 25 = 3t^2 - 16t + 25$.

16. $x^2 + y^2 \geq 9$



17. $\begin{cases} x + y \leq 6 \\ x \geq 2 \\ y \geq 0 \end{cases}$



18. Line through $(0, 0)$ and $(0, 7)$:

$x = 0$

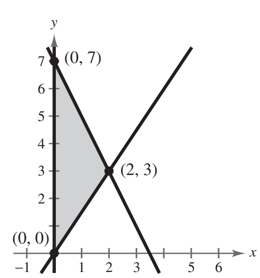
Line through $(0, 0)$ and $(2, 3)$:

$y = \frac{3}{2}x$ or $3x - 2y = 0$

Line through $(0, 7)$ and $(2, 3)$:

$y = -2x + 7$ or $2x + y = 7$

Inequalities: $\begin{cases} x \geq 0 \\ 3x - 2y \leq 0 \\ 2x + y \leq 7 \end{cases}$



19. Vertices: (0, 0), (0, 7), (6, 0), (3, 5)

$z = 30x + 26y$

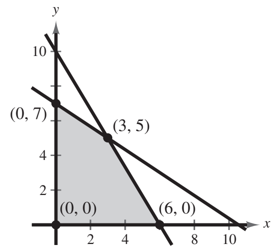
At (0, 0): $z = 0$

At (0, 7): $z = 182$

At (6, 0): $z = 180$

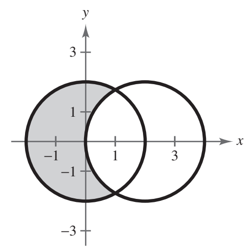
At (3, 5): $z = 220$

The maximum value of z occurs at (3, 5) and is 220.



20. $x^2 + y^2 \le 4$

$(x - 2)^2 + y^2 \ge 4$



21. $\dfrac{1 - 2x}{x^2 + x} = \dfrac{1 - 2x}{x(x + 1)} = \dfrac{A}{x} + \dfrac{B}{x + 1}$

$1 - 2x = A(x + 1) + Bx$

When $x = 0$, $1 = A$.

When $x = -1$, $3 = -B \implies B = -3$.

$\dfrac{1 - 2x}{x^2 + x} = \dfrac{1}{x} - \dfrac{3}{x + 1}$

22. $\dfrac{6x - 17}{(x - 3)^2} = \dfrac{A}{x - 3} + \dfrac{B}{(x - 3)^2}$

$6x - 17 = A(x - 3) + B$

When $x = 3$, $1 = B$.

When $x = 0$, $-17 = -3A + B \implies A = 6$.

$\dfrac{6x - 17}{(x - 3)^2} = \dfrac{6}{x - 3} + \dfrac{1}{(x - 3)^2}$

Chapter 8 Practice Test Solutions

1. $\begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 9 \end{bmatrix}$

$-3R_1 + R_2 \longrightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \end{bmatrix}$

$2R_2 + R_1 \longrightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}$

2. $\begin{cases} 3x + 5y = 3 \\ 2x - y = -11 \end{cases}$

$\begin{bmatrix} 3 & 5 & \vdots & 3 \\ 2 & -1 & \vdots & -11 \end{bmatrix}$

$-R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 6 & \vdots & 14 \\ 2 & -1 & \vdots & -11 \end{bmatrix}$

$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 6 & \vdots & 14 \\ 0 & -13 & \vdots & -39 \end{bmatrix}$

$-\frac{1}{13}R_2 \rightarrow \begin{bmatrix} 1 & 6 & \vdots & 14 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$

$-6R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 0 & \vdots & -4 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$

$x = -4, y = 3$

Solution: $(-4, 3)$

3. $\begin{cases} 2x + 3y = -3 \\ 3x - 2y = 8 \\ x + y = 1 \end{cases}$

$$\begin{bmatrix} 2 & 3 & \vdots & -3 \\ 3 & 2 & \vdots & 8 \\ 1 & 1 & \vdots & 1 \end{bmatrix}$$

$$\begin{matrix} R_3 \rightarrow \\ \\ R_1 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 3 & 2 & \vdots & 8 \\ 2 & 3 & \vdots & -3 \end{bmatrix}$$

$$\begin{matrix} \\ -3R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & -1 & \vdots & 5 \\ 0 & 1 & \vdots & -5 \end{bmatrix}$$

$$\begin{matrix} \\ -R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & 1 & \vdots & -5 \\ 0 & 1 & \vdots & -5 \end{bmatrix}$$

$$\begin{matrix} -R_2 + R_1 \rightarrow \\ \\ -R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & \vdots & 6 \\ 0 & 1 & \vdots & -5 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

$x = 6, y = -5$

Solution: $(6, -5)$

4. $\begin{cases} x + 3z = -5 \\ 2x + y = 0 \\ 3x + y - z = -3 \end{cases}$

$$\begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 2 & 1 & 0 & \vdots & 0 \\ 3 & 1 & -1 & \vdots & 3 \end{bmatrix}$$

$$\begin{matrix} \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 0 & 1 & -6 & \vdots & 10 \\ 0 & 1 & -10 & \vdots & 18 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 0 & 1 & -6 & \vdots & 10 \\ 0 & 0 & -4 & \vdots & 8 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -\frac{1}{4}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 0 & 1 & -6 & \vdots & 10 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

$$\begin{matrix} -3R_3 + R_1 \rightarrow \\ 6R_3 + R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -2 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

$x = 1, y = -2, z = -2$

Solution: $(1, -2, -2)$

5. $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & -7 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} (1)(1) + (4)(0) + (5)(-1) & (1)(6) + (4)(-7) + (5)(2) \\ (2)(1) + (0)(0) + (-3)(-1) & (2)(6) + (0)(-7) + (-3)(2) \end{bmatrix} = \begin{bmatrix} -4 & -12 \\ 5 & 6 \end{bmatrix}$

6. $3A - 5B = 3\begin{bmatrix} 9 & 1 \\ -4 & 8 \end{bmatrix} - 5\begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 27 & 3 \\ -12 & 24 \end{bmatrix} - \begin{bmatrix} 30 & -10 \\ 15 & 25 \end{bmatrix}$

$= \begin{bmatrix} -3 & 13 \\ -27 & -1 \end{bmatrix}$

7. $f(A) = \begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix}^2 - 7\begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix} + 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix} - \begin{bmatrix} 21 & 0 \\ 49 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ 28 & 1 \end{bmatrix} - \begin{bmatrix} 21 & 0 \\ 49 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 \\ -21 & 2 \end{bmatrix}$

8. False since

$(A + B)(A + 3B) = A(A + 3B) + B(A + 3B)$

$= A^2 + 3AB + BA + 3B^2$ and, in general, $AB \neq BA$.

9. $\left[\begin{array}{rr:rr} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$

$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr:rr} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right]$

$2R_2 + R_1 \rightarrow \left[\begin{array}{rr:rr} 1 & 0 & -5 & 2 \\ 0 & -1 & -3 & 1 \end{array}\right]$

$\begin{array}{r} \\ -R_2 \rightarrow \end{array}\left[\begin{array}{rr:rr} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$

$A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

10. $\left[\begin{array}{rrr:rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 6 & 5 & 0 & 1 & 0 \\ 6 & 10 & 8 & 0 & 0 & 1 \end{array}\right]$

$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -6R_1 + R_3 \rightarrow \end{array}\left[\begin{array}{rrr:rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & -3 & 1 & 0 \\ 0 & 4 & 2 & -6 & 0 & 1 \end{array}\right]$

$\begin{array}{r} \\ -R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrr:rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 3 & 1 & -1 \\ 0 & 4 & 2 & -6 & 0 & 1 \end{array}\right]$

$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ 4R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrr:rrr} 1 & 0 & 1 & 4 & 1 & -1 \\ 0 & -1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 2 & 6 & 4 & -3 \end{array}\right]$

$\begin{array}{r} \\ -R_2 \rightarrow \\ \frac{1}{2}R_3 \rightarrow \end{array}\left[\begin{array}{rrr:rrr} 1 & 0 & 1 & 4 & 1 & -1 \\ 0 & 1 & 0 & -3 & -1 & 1 \\ 0 & 0 & 1 & 3 & 2 & -\frac{3}{2} \end{array}\right]$

$\begin{array}{r} -R_3 + R_1 \rightarrow \\ \\ \\ \end{array}\left[\begin{array}{rrr:rrr} 1 & 0 & 0 & 1 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & -3 & -1 & 1 \\ 0 & 0 & 1 & 3 & 2 & -\frac{3}{2} \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1 & -1 & \frac{1}{2} \\ -3 & -1 & 1 \\ 3 & 2 & -\frac{3}{2} \end{bmatrix}$

11. (a) $\begin{cases} x + 2y = 4 \\ 3x + 5y = 1 \end{cases}$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{1}{5 - 6}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -18 \\ 11 \end{bmatrix}$

$x = -18, y = 11$

Solution: $(-18, 11)$

(b) $\begin{cases} x + 2y = 3 \\ 3x + 5y = -2 \end{cases}$

Again, $A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$.

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -19 \\ 11 \end{bmatrix}$

$x = -19, y = 11$

Solution: $(-19, 11)$

12. $\begin{vmatrix} 6 & -1 \\ 3 & 4 \end{vmatrix} = 24 - (-3) = 27$

13. $\begin{vmatrix} 1 & 3 & -1 \\ 5 & 9 & 0 \\ 6 & 2 & -5 \end{vmatrix} = -1\begin{vmatrix} 5 & 9 \\ 6 & 2 \end{vmatrix} - 5\begin{vmatrix} 1 & 3 \\ 5 & 9 \end{vmatrix} = -(-44) - 5(-6) = 74$

14. Expand along Row 2.

$$\begin{vmatrix} 1 & 4 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 3 & 5 & -1 & 1 \\ 2 & 0 & 6 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ 2 & 6 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 4 & 3 \\ 3 & 5 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= 51 + 2(-29) = -7$$

15. $\begin{vmatrix} 6 & 4 & 3 & 0 & 6 \\ 0 & 5 & 1 & 4 & 8 \\ 0 & 0 & 2 & 7 & 3 \\ 0 & 0 & 0 & 9 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 6\begin{vmatrix} 5 & 1 & 4 & 8 \\ 0 & 2 & 7 & 3 \\ 0 & 0 & 9 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 6(5)\begin{vmatrix} 2 & 7 & 3 \\ 0 & 9 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 6(5)(2)\begin{vmatrix} 9 & 2 \\ 0 & 1 \end{vmatrix} = 6(5)(2)(9) = 540$

16. $\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 7 & 1 \\ 5 & 0 & 1 \\ 3 & 9 & 1 \end{vmatrix} = \frac{1}{2}(31) = \frac{31}{2}$

17. $\begin{vmatrix} x & y & 1 \\ 2 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 3x - 3y + 15 = 0$ or, equivalently, $x - y + 5 = 0$

18. $x = \dfrac{\begin{vmatrix} 4 & -7 \\ 11 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -7 \\ 2 & 5 \end{vmatrix}} = \dfrac{97}{44}$

19. $z = \dfrac{\begin{vmatrix} 3 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 0 & 1 \\ 0 & 1 & 4 \\ 1 & -1 & 0 \end{vmatrix}} = \dfrac{14}{11}$

20. $y = \dfrac{\begin{vmatrix} 721.4 & 33.77 \\ 45.9 & 19.85 \end{vmatrix}}{\begin{vmatrix} 721.4 & -29.1 \\ 45.9 & 105.6 \end{vmatrix}} = \dfrac{12{,}769.747}{77{,}515.530} \approx 0.1647$

Chapter 9 Practice Test Solutions

1. $a_n = \dfrac{2n}{(n+2)!}$

$a_1 = \dfrac{2(1)}{3!} = \dfrac{2}{6} = \dfrac{1}{3}$

$a_2 = \dfrac{2(2)}{4!} = \dfrac{4}{24} = \dfrac{1}{6}$

$a_3 = \dfrac{2(3)}{5!} = \dfrac{6}{120} = \dfrac{1}{20}$

$a_4 = \dfrac{2(4)}{6!} = \dfrac{8}{720} = \dfrac{1}{90}$

$a_5 = \dfrac{2(5)}{7!} = \dfrac{10}{5040} = \dfrac{1}{504}$

Terms: $\frac{1}{3}, \frac{1}{6}, \frac{1}{20}, \frac{1}{90}, \frac{1}{504}$

2. $a_n = \dfrac{n+3}{3^n}$

3. $\displaystyle\sum_{i=1}^{6}(2i - 1) = 1 + 3 + 5 + 7 + 9 + 11 = 36$

4. $a_1 = 23,\ d = -2$

$a_2 = 23 + (-2) = 21$

$a_3 = 21 + (-2) = 19$

$a_4 = 19 + (-2) = 17$

$a_5 = 17 + (-2) = 15$

Terms: 23, 21, 19, 17, 15

5. $a_1 = 12, d = 3, n = 50$

$a_n = a_1 + (n - 1)d$

$a_{50} = 12 + (50 - 1)3 = 159$

6. $a_1 = 1$

$a_{200} = 200$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{200} = \frac{200}{2}(1 + 200) = 20{,}100$

7. $a_1 = 7,\ r = 2$

$a_2 = 7(2) = 14$

$a_3 = 7(2)^2 = 28$

$a_4 = 7(2)^3 = 56$

$a_5 = 7(2)^4 = 112$

Terms: 7, 14, 28, 56, 112

8. $\sum_{n=1}^{10} 6\left(\frac{2}{3}\right)^{n-1}, a_1 = 6, r = \frac{2}{3}, n = 10$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} = \frac{6\left[1 - \left(\frac{2}{3}\right)^{10}\right]}{1 - \frac{2}{3}} = 18\left(1 - \frac{1024}{59{,}049}\right) = \frac{116{,}050}{6561} \approx 17.6879$$

9. $\sum_{n=0}^{\infty} (0.03)^n = \sum_{n=1}^{\infty} (0.03)^{n-1}, a_1 = 1, r = 0.03$

$$S = \frac{a_1}{1 - r} = \frac{1}{1 - 0.03} = \frac{1}{0.97} = \frac{100}{97} \approx 1.0309$$

10. For $n = 1, 1 = \frac{1(1 + 1)}{2}$.

Assume that $S_k = 1 + 2 + 3 + 4 + \cdots + k = \frac{k(k + 1)}{2}$.

Then
$$S_{k+1} = 1 + 2 + 3 + 4 + \cdots + k + (k + 1) = \frac{k(k + 1)}{2} + k + 1 = \frac{k(k + 1)}{2} + \frac{2(k + 1)}{2} = \frac{(k + 1)(k + 2)}{2}.$$

Thus, by the principle of mathematical induction, $1 + 2 + 3 + 4 + \cdots + n = \frac{n(n + 1)}{2}$ for all integers $n \ge 1$.

11. For $n = 4, 4! > 2^4$. Assume that $k! > 2^k$.

Then $(k + 1)! = (k + 1)(k!) > (k + 1)2^k > 2 \cdot 2^k = 2^{k+1}$.

Thus, by the extended principle of mathematical induction, $n! > 2^n$ for all integers $n \ge 4$.

12. ${}_{13}C_4 = \dfrac{13!}{(13-4)!4!} = 715$

13. $(x+3)^5 = x^5 + 5x^4(3) + 10x^3(3)^2 + 10x^2(3)^3 + 5x(3)^4 + (3)^5$
$= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$

14. $-{}_{12}C_5 x^7(2)^5 = -25{,}344x^7$

15. ${}_{30}P_4 = \dfrac{30!}{(30-4)!} = 657{,}720$

16. $6! = 720$ ways

17. ${}_{12}P_3 = 1320$

18. $P(2) + P(3) + P(4) = \dfrac{1}{36} + \dfrac{2}{36} + \dfrac{3}{36} = \dfrac{6}{36} = \dfrac{1}{6}$

19. $P(K, B10) = \dfrac{4}{52} \cdot \dfrac{2}{51} = \dfrac{2}{663}$

20. Let A = probability of no faulty units.

$P(A) = \left(\dfrac{997}{1000}\right)^{50} \approx 0.8605$

$P(A') = 1 - P(A) \approx 0.1395$

Chapter 10 Practice Test Solutions

1. $3x + 4y = 12 \implies y = -\dfrac{3}{4}x + 3 \implies m_1 = -\dfrac{3}{4}$

$4x - 3y = 12 \implies y = \dfrac{4}{3}x - 4 \implies m_2 = \dfrac{4}{3}$

$\tan\theta = \left|\dfrac{(4/3) - (-3/4)}{1 + (4/3)(-3/4)}\right| = \left|\dfrac{25/12}{0}\right|$

Since $\tan\theta$ is undefined, the lines are perpendicular (note that $m_2 = -1/m_1$) and $\theta = 90°$.

2. $x_1 = 5, x_2 = -9, A = 3, B = -7, C = -21$

$d = \dfrac{|3(5) + (-7)(-9) + (-21)|}{\sqrt{3^2 + (-7)^2}} = \dfrac{57}{\sqrt{58}} \approx 7.484$

3. $x^2 - 6x - 4y + 1 = 0$

$x^2 - 6x + 9 = 4y - 1 + 9$

$(x-3)^2 = 4y + 8$

$(x-3)^2 = 4(1)(y+2) \implies p = 1$

Vertex: $(3, -2)$

Focus: $(3, -1)$

Directrix: $y = -3$

4. Vertex: $(2, -5)$

Focus: $(2, -6)$

Vertical axis; opens downward with $p = -1$

$(x-h)^2 = 4p(y-k)$

$(x-2)^2 = 4(-1)(y+5)$

$x^2 - 4x + 4 = -4y - 20$

$x^2 - 4x + 4y + 24 = 0$

5. $x^2 + 4y^2 - 2x + 32y + 61 = 0$

$$(x^2 - 2x + 1) + 4(y^2 + 8y + 16) = -61 + 1 + 64$$
$$(x - 1)^2 + 4(y + 4)^2 = 4$$
$$\frac{(x - 1)^2}{4} + \frac{(y + 4)^2}{1} = 1$$

$a = 2, b = 1, c = \sqrt{3}$

Horizontal major axis

Center: $(1, -4)$

Foci: $(1 \pm \sqrt{3}, -4)$

Vertices: $(3, -4), (-1, -4)$

Eccentricity: $e = \frac{\sqrt{3}}{2}$

6. Vertices: $(0, \pm 6)$

Eccentricity: $e = \frac{1}{2}$

Center: $(0, 0)$

Vertical major axis

$a = 6, e = \frac{c}{a} = \frac{c}{6} = \frac{1}{2} \Rightarrow c = 3$

$b^2 = (6)^2 - (3)^2 = 27$

$$\frac{x^2}{27} + \frac{y^2}{36} = 1$$

7. $16y^2 - x^2 - 6x - 128y + 231 = 0$

$$16(y^2 - 8y + 16) - (x^2 + 6x + 9) = -231 + 256 - 9$$
$$16(y - 4)^2 - (x + 3)^2 = 16$$
$$\frac{(y - 4)^2}{1} - \frac{(x + 3)^2}{16} = 1$$

$a = 1, b = 4, c = \sqrt{17}$

Center: $(-3, 4)$

Vertical transverse axis

Vertices: $(-3, 5), (-3, 3)$

Foci: $(-3, 4 \pm \sqrt{17})$

Asymptotes: $y = 4 \pm \frac{1}{4}(x + 3)$

8. Vertices: $(\pm 3, 2)$

Foci: $(\pm 5, 2)$

Center: $(0, 2)$

Horizontal transverse axis

$a = 3, c = 5, b = 4$

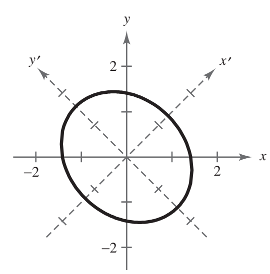
$$\frac{(x - 0)^2}{9} - \frac{(y - 2)^2}{16} = 1$$
$$\frac{x^2}{9} - \frac{(y - 2)^2}{16} = 1$$

9. $5x^2 + 2xy + 5y^2 - 10 = 0$

$A = 5, B = 2, C = 5$

$\cot 2\theta = \frac{5 - 5}{2} = 0$

$2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$



$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$

$= \frac{x' - y'}{\sqrt{2}}$

$x = x' \cos \frac{\pi}{4} + y' \sin \frac{\pi}{4}$

$= \frac{x' + y'}{\sqrt{2}}$

$$5\left(\frac{x' - y'}{\sqrt{2}}\right)^2 + 2\left(\frac{x' - y'}{\sqrt{2}}\right)\left(\frac{x' + y'}{\sqrt{2}}\right) + 5\left(\frac{x' + y'}{\sqrt{2}}\right)^2 - 10 = 0$$
$$\frac{5(x')^2}{2} - \frac{10x'y'}{2} + \frac{5(y')^2}{2} + (x')^2 - (y')^2 + \frac{5(x')^2}{2} + \frac{10x'y'}{2} + \frac{5(y')^2}{2} - 10 = 0$$
$$6(x')^2 + 4(y')^2 - 10 = 0$$
$$\frac{3(x')^2}{5} + \frac{2(y')^2}{5} = 1$$
$$\frac{(x')^2}{5/3} + \frac{(y')^2}{5/2} = 1$$

Ellipse centered at the origin

10. (a) $6x^2 - 2xy + y^2 = 0$

$A = 6, B = -2, C = 1$

$B^2 - 4AC = (-2)^2 - 4(6)(1) = -20 < 0$

Ellipse

(b) $x^2 + 4xy + 4y^2 - x - y + 17 = 0$

$A = 1, B = 4, C = 4$

$B^2 - 4AC = (4)^2 - 4(1)(4) = 0$

Parabola

11. Polar: $\left(\sqrt{2}, \frac{3\pi}{4}\right)$

$x = \sqrt{2}\cos\frac{3\pi}{4} = \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = -1$

$y = \sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 1$

Rectangular: $(-1, 1)$

12. Rectangular: $\left(\sqrt{3}, -1\right)$

$r = \pm\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = \pm 2$

$\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$

Polar: $\left(-2, \frac{2\pi}{3}\right)$ or $\left(2, \frac{5\pi}{3}\right)$

13. Rectangular: $4x - 3y = 12$

Polar: $4r\cos\theta - 3r\sin\theta = 12$

$r(4\cos\theta - 3\sin\theta) = 12$

$$r = \frac{12}{4\cos\theta - 3\sin\theta}$$

14. Polar: $r = 5\cos\theta$

$r^2 = 5r\cos\theta$

Rectangular: $x^2 + y^2 = 5x$

$x^2 + y^2 - 5x = 0$

15. $r = 1 - \cos\theta$

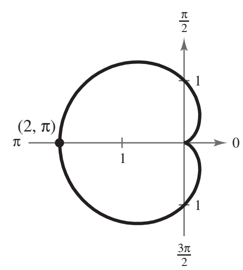
Cardioid

Symmetry: Polar axis

Maximum value of $|r|$: $r = 2$ when $\theta = \pi$

Zero of r: $r = 0$ when $\theta = 0$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	0	1	2	1



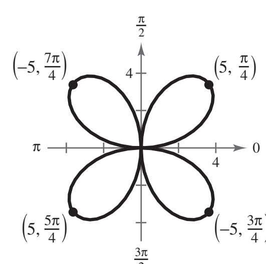
16. $r = 5\sin 2\theta$

Rose curve with four petals

Symmetry: Polar axis, $\theta = \frac{\pi}{2}$, and pole

Maximum value of $|r|$: $|r| = 5$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Zeros of r: $r = 0$ when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

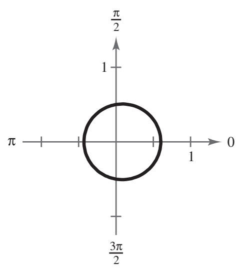


17. $r = \dfrac{3}{6 - \cos\theta}$

$r = \dfrac{1/2}{1 - (1/6)\cos\theta}$

$e = \dfrac{1}{6} < 1$, so the graph is an ellipse.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{3}{7}$	$\frac{1}{2}$



18. Parabola

Vertex: $\left(6, \dfrac{\pi}{2}\right)$

Focus: $(0, 0)$

$e = 1$

$r = \dfrac{ep}{1 + e\sin\theta}$

$r = \dfrac{p}{1 + \sin\theta}$

$6 = \dfrac{p}{1 + \sin(\pi/2)}$

$6 = \dfrac{p}{2}$

$12 = p$

$r = \dfrac{12}{1 + \sin\theta}$

19. $x = 3 - 2\sin\theta, y = 1 + 5\cos\theta$

$\dfrac{x-3}{-2} = \sin\theta, \dfrac{y-1}{5} = \cos\theta$

$\left(\dfrac{x-3}{-2}\right)^2 + \left(\dfrac{y-1}{5}\right)^2 = 1$

$\dfrac{(x-3)^2}{4} + \dfrac{(y-1)^2}{25} = 1$

20. $x = e^{2t}, y = e^{4t}$

$x > 0, y > 0$

$y = (e^{2t})^2 = (x)^2 = x^2, x > 0, y > 0$

Chapter 11 Practice Test Solutions

1. Let $A = (0, 0, 0)$, $B = (1, 2, -4)$, $C = (0, -2, -1)$.

Side AB: $\sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$

Side AC: $\sqrt{0^2 + 2^2 + 1^2} = \sqrt{5}$

Side BC: $\sqrt{(-1)^2 + (-2-2)^2 + (-1+4)^2} = \sqrt{1 + 16 + 9} = \sqrt{26}$

$BC^2 = AB^2 + AC^2$

$26 = 21 + 5$

2. $(x - 0)^2 + (y - 4)^2 + (z - 1)^2 = 5^2$

$x^2 + (y - 4)^2 + (z - 1)^2 = 25$

3. $(x^2 + 2x + 1) + y^2 + (z^2 - 4z + 4) = 1 + 4 + 11$

$(x + 1)^2 + y^2 + (z - 2)^2 = 16$

Center: $(-1, 0, 2)$

Radius: 4

4. $\mathbf{u} - 3\mathbf{v} = \langle 1, 0, -1\rangle - 3\langle 4, 3, -6\rangle$
$= \langle 1, 0, -1\rangle - \langle 12, 9, -18\rangle$
$= \langle -11, -9, 17\rangle$

5. $\frac{1}{2}\mathbf{v} = \frac{1}{2}\langle 2, 4, -6\rangle = \langle 1, 2, -3\rangle$
$\left\|\frac{1}{2}\mathbf{v}\right\| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

6. $\mathbf{u} \cdot \mathbf{v} = \langle 2, 1, -3\rangle \cdot \langle 1, 1, -2\rangle$
$= 2 + 1 + 6 = 9$

7. Because $\mathbf{v} = \langle -3, -3, 3\rangle = -3\langle 1, 1, -1\rangle = -3\mathbf{u}$, $\mathbf{u}$ and $\mathbf{v}$ are parallel.

8. $$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 0 & 2 \\ 1 & -1 & 3 \end{vmatrix} = \langle 2, 5, 1\rangle$$
$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = \langle -2, -5, -1\rangle$

9. $$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 4 \end{vmatrix}$$
$= 1(-4) - 1(-1) + 1(1)$
$= -4 + 1 + 1 = -2$
Volume $= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |-2| = 2$

10. $\mathbf{v} = \langle (2 - 0), -3 - (-3), 4 - 3\rangle = \langle 2, 0, 1\rangle$
$x = 2 + 2t, y = -3, z = 4 + t$

11. $1(x - 1) - 1(y - 2) + 0(z - 3) = 0$
$x - 1 - y + 2 = 0$
$x - y + 1 = 0$

12. $\overrightarrow{AB} = \langle 1, 1, 1\rangle, \overrightarrow{AC} = \langle 1, 2, 3\rangle$
$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 1, -2, 1\rangle$$
Plane: $1(x - 0) - 2(y - 0) + (z - 0) = 0$
$x - 2y + z = 0$

13. $\mathbf{n}_1 = \langle 1, 1, -1\rangle, \mathbf{n}_2 = \langle 3, -4, -1\rangle$
$\mathbf{n}_1 \cdot \mathbf{n}_2 = 3 - 4 + 1 = 0 \implies$ Orthogonal planes

14. $\mathbf{n} = \langle 1, 2, 1\rangle, Q = (1, 1, 1), P = (0, 0, 6)$ on plane, $\overrightarrow{PQ} = \langle 1, 1, -5\rangle$
$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|1 + 2 - 5|}{\sqrt{1 + 4 + 1}} = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$$

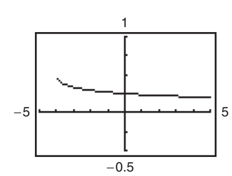
Chapter 12 Practice Test Solutions

1.

x	2.9	2.99	3	3.01	3.1
$f(x)$	0.1695	0.1669	?	0.1664	0.1639

$$\lim_{x\to 3} \frac{x - 3}{x^2 - 9} \approx 0.1667$$

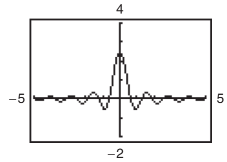
2. $$\lim_{x\to 0} \frac{\sqrt{x + 4} - 2}{x} \approx \frac{1}{4}$$



3. $\lim_{x\to 2} e^{x-2} = e^{2-2} = e^0 = 1$

4. $$\lim_{x\to 1} \frac{x^3 - 1}{x - 1} = \lim_{x\to 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$
$$= \lim_{x\to 1} (x^2 + x + 1) = 3$$

5. $\lim_{x \to 0} \frac{\sin 5x}{2x} \approx 2.5$



6. The limit does not exist. If

$$f(x) = \frac{|x + 2|}{x + 2},$$

then $f(x) = 1$ for $x > -2$, and $f(x) = -1$ for $x < -2$.

7.
$$\begin{aligned} m_{\sec} &= \frac{f(4 + h) - f(4)}{h} \\ &= \frac{\sqrt{4 + h} - 2}{h} \\ &= \frac{\sqrt{4 + h} - 2}{h} \cdot \frac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2} \\ &= \frac{(4 + h) - 4}{h\left[\sqrt{4 + h} + 2\right]} \\ &= \frac{h}{h\left[\sqrt{4 + h} + 2\right]} \\ &= \frac{1}{\sqrt{4 + h} + 2}, \quad h \neq 0 \end{aligned}$$

$$m = \lim_{h \to 0} \frac{1}{\sqrt{4 + h} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$$

8.
$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{[3(x + h) - 1] - [3x - 1]}{h} \\ &= \lim_{h \to 0} \frac{3x + 3h - 1 - 3x + 1}{h} \\ &= \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3 \end{aligned}$$

9. (a) $\lim_{x \to \infty} \frac{3}{x^4} = 0$

(b) $\lim_{x \to -\infty} \frac{x^2}{x^2 + 3} = 1$

(c) $\lim_{x \to \infty} \frac{|x|}{1 - x} = -1$

10. $a_1 = 0,\ a_2 = \frac{1 - 4}{8 + 1} = -\frac{1}{3},\ a_3 = \frac{1 - 9}{18 + 1} = -\frac{8}{19},$

$a_4 = \frac{1 - 16}{33} = -\frac{15}{33}$

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1 - n^2}{2n^2 + 1} = -\frac{1}{2}$

11. $\sum_{i=1}^{25} i^2 + \sum_{i=1}^{25} i = \frac{25(26)(51)}{6} + \frac{25(26)}{2} = \frac{25(26)}{6}[51 + 3] = \frac{25(26)(54)}{6} = 5850$

12. $\sum_{i=1}^{n} \frac{i^2}{n^3} = \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \frac{1}{n^3}\left[\frac{n(n + 1)(2n + 1)}{6}\right] = \frac{2n^2 + 3n + 1}{6n^2} = S(n)$

$\lim_{n \to \infty} S(n) = \frac{1}{3}$

13. Width of rectangles: $\frac{b - a}{n} = \frac{1}{n}$

Height: $f\left(a + \frac{(b - a)i}{n}\right) = f\left(\frac{i}{n}\right) = 1 - \left(\frac{i}{n}\right)^2$

$A \approx \sum_{i=1}^{n} \left[1 - \frac{i^2}{n^2}\right]\frac{1}{n} = \sum_{i=1}^{n} \frac{1}{n} - \sum_{i=1}^{n} \frac{i^2}{n^3} = 1 - \frac{1}{n^3} \frac{n(n + 1)(2n + 1)}{6}$

$A = \lim_{n \to \infty} A_n = 1 - \frac{1}{3} = \frac{2}{3}$