Adapting this for $(x^2 + 2x^{-1})^6$ where $a = x$, $b = 2x^{-1}$, then

$(x^2 + 2x^{-1})^6$

$= (x^2)^6 + 6(x^2)^5.(2x^{-1}) + 15(x^2)^4.(2x^{-1})^2 + 20(x^2)^3.(2x^{-1})^3 + 15(x^2)^2.(2x^{-1})^4$
$\quad + 6(x^2)^1.(2x^{-1})^5 + (2x^{-1})^6$

$= x^{12} + 6(x^{10}).(2x^{-1}) + 15(x^8).(4x^{-2}) + 20(x^6).(8x^{-3}) + 15(x^4).(16x^{-4})$
$\quad + 6(x^2).(32x^{-5}) + 64x^{-6}$

$= x^{12} + 12x^9 + 60x^6 + 160x^3 + 240 + 192x^{-3} + 64x^{-6}$ #

EXERCISES SET 8A.

Make up your own Pascal's Triangle set, for values of n from n = 0 to n = 10. Keep this in a prominent place for use in the following exercises.

1. USING THE PASCAL TRIANGLE BINOMIAL COEFFICIENTS, GIVE THE EXPANSIONS OF THE FOLLOWING:

(a) $(1 + x)^4$ (b) $(1 - x)^4$ (c) $(1 + y)^6$ (d) $(1 + \frac{1}{5}x)^3$

(e) $(1 + \frac{2x}{y})^4$ (f) $(1 - 2x)^5$ †(g) $(1 + \frac{x}{2})^8$ †(h) $(1 - \frac{2}{x})^7$

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2. IN THE EXPANSION OF $(1 + x)^8$, FIND THE

(i) fifth term (u_5) (ii) sixth term (u_6) (iii) ratio $u_5:u_6$

What are these values when $x = 2$?

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3. FOR THE EXPANSIONS $(1 + x)^7$ AND $(1 + x)^6$ SHOW THAT

(a) the sum of the coefficients of the first expansion is double that of the second expansion;

(b) the ratio of the products of these coefficients is $7^6:6!$ (where $6! = 6.5.4.3.2.1$).

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4. (i) EXPAND OUT $(1 + 2x)^5$ and hence find the coefficient of x^2 in the expansion of $(1 + x)^2(1 + 2x)^5$.

†(ii) DERIVE THE EXPANSIONS OF $(1 + x)^6$ AND $(1 + x^{-2})^4$. Hence, determine the value of the term independent of x in the expansion $(1 + x)^6(1 + x^{-2})^4$.

---

5. (i) IF x IS SO SMALL THAT THIRD AND HIGHER POWERS MAY BE NEGLECTED, then $(3 + 2x)(1 + x)^4 = a + bx + cx^2$, where a, b, c are certain constants. Find a, b, c.

(ii) IF $(1 + \sqrt{2})^5 = a + b\sqrt{2}$, FIND THE VALUES OF THE INTEGERS a, b.

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†6. (i) WRITE DOWN THE CONDITION FOR THE NUMBERS p, q, r TO FORM
(a) an arithmetic sequence (b) a geometric sequence

(ii) IN THE EXPANSION OF $(1 + x)^7$ IN ASCENDING POWERS OF x, it is known that the
(a) second, third and fourth terms form an arithmetic progression; find the possible values of x;
(b) second, third and fifth terms form a geometric progression, find x.

---

7. (i) FOR WHAT VALUE OF k WILL THE COEFFICIENTS OF x^3 AND x^4 in the expansion of $(1 + kx)^8$ be in the ratio 2 : 3.

(ii) SHOW THAT $(1 - x)^6 + (1 + x)^6 = 2\{1 + 15x^2 + 15x^4 + x^6\}$, and hence prove this is rational when $x = \sqrt{3}$. What is its value?

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8. EXPAND $(1 - 2x)^6$ IN ASCENDING POWERS OF x. Hence, calculate the value of $(0.98)^6$ correct to 5 places of decimals.
Check your result by use of calculator.

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†9. WRITE DOWN AS DECIMALS ALL TERMS IN THE EXPANSION OF $(1 + \frac{2}{100})^5$, and hence find, correct to the nearest cent, the compound interest (\$ I) on \$1000 at 2% p.a. for 5 years.
{*Hint: The amount \$ A to which a principal of \$ P grows at r% p.a. C.I. over n years, interest paid annually, is given by* $A = P(1 + \frac{r}{100})^n$}.
Compare this result with the answer from tables or calculator.

---

10. EXPAND OUT THE FOLLOWING, USING THE PASCAL TRIANGLE to give the appropriate binomial coefficients:

(a) $(x + y)^3$ (b) $(p - q)^8$ (c) $(2x - b)^7$ (d) $(x + \frac{1}{x})^5$
(e) $(x - 5x^{-2})^3$ †(f) $(\frac{x}{2} - 2)^6$ †(g) $(x^{\frac{1}{2}} + 2y^{1/3})^4$ †(h) $(2x^{-1} + \frac{1}{3}x^2)^5$

---

11. (i) IN THE EXPANSION OF $(b + 2x)^6$ in ascending powers of x, the coefficients of x and x^2 are equal; find b.

(ii) FIND THE EXPANSION OF $(2 + x)^4 - (2 - x)^4$, and hence prove that $(2 + \sqrt{3})^4 - (2 - \sqrt{3})^4 = 112\sqrt{3}$.

---

12. (i) EXPAND $(2 - \frac{1}{x})^3$ AND HENCE DETERMINE the term independent of x in the expansion $(2 + 3x - 4x^2)(2 - \frac{1}{x})^3$.

SET 8A (PAGE 274)

1. (a) $1 + 4x + 6x^2 + 4x^3 + x^4$ (b) $1 - 4x + 6x^2 - 4x^3 + x^4$
(c) $1 + 6y + 15y^2 + 20y^3 + 15y^4 + 6y^5 + y^6$ (d) $1 + 3x/5 + 3x^2/25 + x^3/125$
(e) $1 + 8x/y + 24x^2/y^2 + 32x^3/y^3 + 16x^4/y^4$
(f) $1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$
(g) $1 + 4x + 7x^2 + 7x^3 + 35x^4/8 + 7x^5/4 + 7x^6/16 + x^7/16 + x^8/256$
(h) $1 - 14/x + 84/x^2 - 280/x^3 + 560/x^4 - 672/x^5 + 448/x^6 - 128/x^7$

2. (i) $70x^4$ (ii) $56x^5$ (iii) $5/(4x)$; 1120; 1792; 5:8

4. (i) $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$; coefft. 61 (ii) term indep. of x is $(1 + 60 + 90 + 4) = 155$

5. (i) $a = 3$, $b = 14$, $c = 26$ (ii) $a = 41$, $b = 29$

6. (i) (a) $q = \frac{1}{2}(p + r)$ (b) $q^2 = pr$ (ii) (a) $x = 1/5$ or 1 $(x \neq 0)$
(b) $x = 9/5$ $(x \neq 0)$

7. (i) $k = 6/5$ (ii) 416

8. $1 - 12x + 60x^2 - 160x^3 + 240x^4 - 192x^5 + 64x^6$; 0.88584; by calculator 0.88584

9. 1 + 0.01 + 0.004 + 0.000 08 + 0.000 000 80 + 0.000 000 003 2; \$104.08; by calculator \$104.08; by tables \$104.00

10. (a) $x^3 + 3x^2y + 3xy^2 + y^3$
(b) $p^8 - 8p^7q + 28p^6q^2 - 56p^5q^3 + 70p^4q^4 - 56p^3q^5 + 28p^2q^6 - 8pq^7 + q^8$
(c) $128x^7 - 448x^6b + 672x^5b^2 - 560x^4b^3 + 280x^3b^4 - 84x^2b^5 + 14xb^6 - b^7$
(d) $x^5 + 5x^3 + 10x + 10x^{-1} + 5x^{-3} + x^{-5}$ (e) $x^3 - 15 + 75x^{-3} - 125x^{-6}$
(f) $x^6/64 - 3x^5/8 + 15x^4/4 - 20x^3 + 60x^2 - 96x + 64$
(g) $x^2 + 8x^{3/2}y^{1/3} + 24xy^{2/3} + 32x^{1/2}y + 16y^{4/3}$
(h) $32x^{-5} + 80x^{-2}/3 + 80x/9 + 40x^4/27 + 10x^7/81 + x^{10}/243$