

The third instalment of \$M is due at the end of the third month, and hence its present worth is  $\$M(1.015)^{-3}$ , and so on.

THE SUM OF THESE PRESENT WORTHS (IN DOLLARS)

$$= M(1.015)^{-1} + M(1.015)^{-2} + M(1.015)^{-3} + \dots \text{ for 3 years i.e. 36 months}$$

This is the sum to 36 terms of a geometric series in which the first term  $a = M(1.015)^{-1}$  and the common ratio  $r = (1.015)^{-1}$

Using  $S_n = a(1 - r^n)/(1 - r)$ , then

$$S_{36} = \frac{M(1.015)^{-1}\{1 - (1.015)^{-36}\}}{1 - (1.015)^{-1}} = \frac{M\{1 - (1.015)^{-36}\}}{0.015} \text{ on multiplying}$$

numerator and denominator by  $(1.015)^1$  and noting  $(1.015)^0 = 1$ .

NOW THE AMOUNT OF THE LOAN IS \$5000; THIS IS THE VALUE OF THE SUM OF THE PRESENT WORTHS.

$$\text{i.e. } \frac{M\{1 - (1.015)^{-36}\}}{0.015} = 5000, \text{ i.e. } M = \frac{5000(0.015)}{1 - (1.015)^{-36}}, \text{ and by calculator}$$

to the nearest c,  $M = 180.76$ .

That is, the equal monthly instalments are \$180.76.#

Note. The loan of \$5000 was charged interest at the rate of 1.5% per month, reducible interest (equivalent to  $12 \times 1.5\% = 18\%$  p.a.).

However, if we consider the loan as though simple not reducible (compound) interest were involved, then we see that for the loan of \$5000, the borrower paid  $36 \times \$180.76$  (this includes interest) over the 36 months.

Thus, over the 3 years of the loan of \$5000, the amount of interest paid was  $\$(36 \times 180.76 - 5000) = \$1507.36$ . Using simple interest formula

$$r = \frac{100I}{Pn} = \frac{100 \times 1507.36}{5000 \times 3} \doteq 10.05\% \text{ p.a.}$$

This rate 10.05% p.a. is the rate usually published by financial institutions, rather than the reducible rate 18% p.a.

#### \*SET 9M

1. A MAN BORROWS \$10000 AT 15% p.a. REDUCIBLE INTEREST, AND PAYS IT OFF IN equal annual instalments over a period of 12 years.

What should his instalments be (to the nearest cent)?

Determine the rate % p.a. simple interest charged on the loan, correct to 2 decimal places.

2. A FINANCIAL INSTITUTION ADVANCES \$4000 AND AGREES TO ACCEPT PAYMENT OF the loan and interest at the rate of 1.5% per month in equal monthly instalments over 2 years.

Find the amount of each instalment (to the nearest c) and calculate the rate (correct to 2 significant figures) which the institution would publish as the rate charged on this loan. (*This is the simple interest rate.*)

3. A WOMAN PROPOSES TO BUY A HOUSE, OF WHICH THE CASH PRICE IS \$100000 by 20 equal annual instalments, the first payment to be made 12 months after the sale. If interest is charged on the loan needed at 9% p.a., find the amount of each instalment.

4. THE CASH PRICE OF A HOUSE IS \$80000. A COUPLE PAY A DEPOSIT OF \$10000 and borrow the balance to buy the house. Interest is charged on the loan at 10% reducible, and the loan (including interest) is to be paid off by equal annual instalments over 30 years, the first instalment to be made 1 year after the time of sale.

Calculate the amount of each instalment (to nearest c).

5. A MAN WISHES TO HAVE \$20000 CAPITAL IN 15 YEARS' TIME. HE PROPOSES TO make a finance company 15 equal annual payments commencing now (the last payment being made 1 year before the \$20000 is to be paid to him). Assuming interest is paid by the company at 12% p.a., find what should be the amount of each payment (to nearest \$).

6. A WOMAN WISHES TO LEAVE \$250000 TO HER DEPENDANTS ON HER DEATH. SHE proposes to make an insurance company equal annual payments commencing now and going on until her death. What should be the amount of the annual payment, assuming interest at 7% p.a.? (*Assume the woman dies in 30 years, immediately after the last payment.*)

7. A MAN BORROWS \$20000 AT 15% p.a. COMPOUND INTEREST. IF THE PRINCIPAL and interest are to be paid by 20 equal annual instalments find approximately (to the nearest c) the amount of each of these.

*To what simple interest rate is this equivalent?*

8. A CORPORATION BORROWS \$50000 TO BE REPAYED WITH INTEREST AT 1% PER MONTH in equal monthly instalments over 4 years.

What sum (to the nearest dollar) must be repaid each month?

9. A MAN BORROWS \$1000 AND AGREES TO REPAY IT BY 10 EQUAL MONTHLY PAYMENTS, the first payment being made at the end of one month and interest being charged at the rate of 2% per month, calculated monthly on that part of the sum which is not yet repaid. Find the value of the monthly payments, to the nearest cent.

*To what simple interest is this loan equivalent?*

10. A SUM OF \$24000 IS BORROWED NOW AT THE RATE OF 15% p.a. REDUCIBLE INTEREST. Payment is made by n equal annual instalments of \$3840 beginning at the end of a year.

Show that  $1 - (1.15)^{-n} = 0.9375$ , and hence find n.